The Simplex Algorithm as a Method to Solve Linear Programming Problems

Linear Programming Problem

Standard Maximization problem in Standard Form	$x_1 + 2x_2 \le 10$ $3x_1 + 2x_2 \le 18$ $x_1, x_2 \ge 0$ $\text{Maximize}: P = 20x_1 + 30x_2$	Decision variables: $x_1, x_2 \ldots$ Constraints ($a_1x_1 + a_2x_2 + \ldots \le b_1$ where $b_n \ge 0$) Non-zero constraints ($x_1, x_2 \ldots \ge 0$) Objective function P
Fundamental Theorem	If an optimum occurs, it will occur at one of the corner points of the feasible region, or along all points of a segment whose endpoints are corner points of the region.	

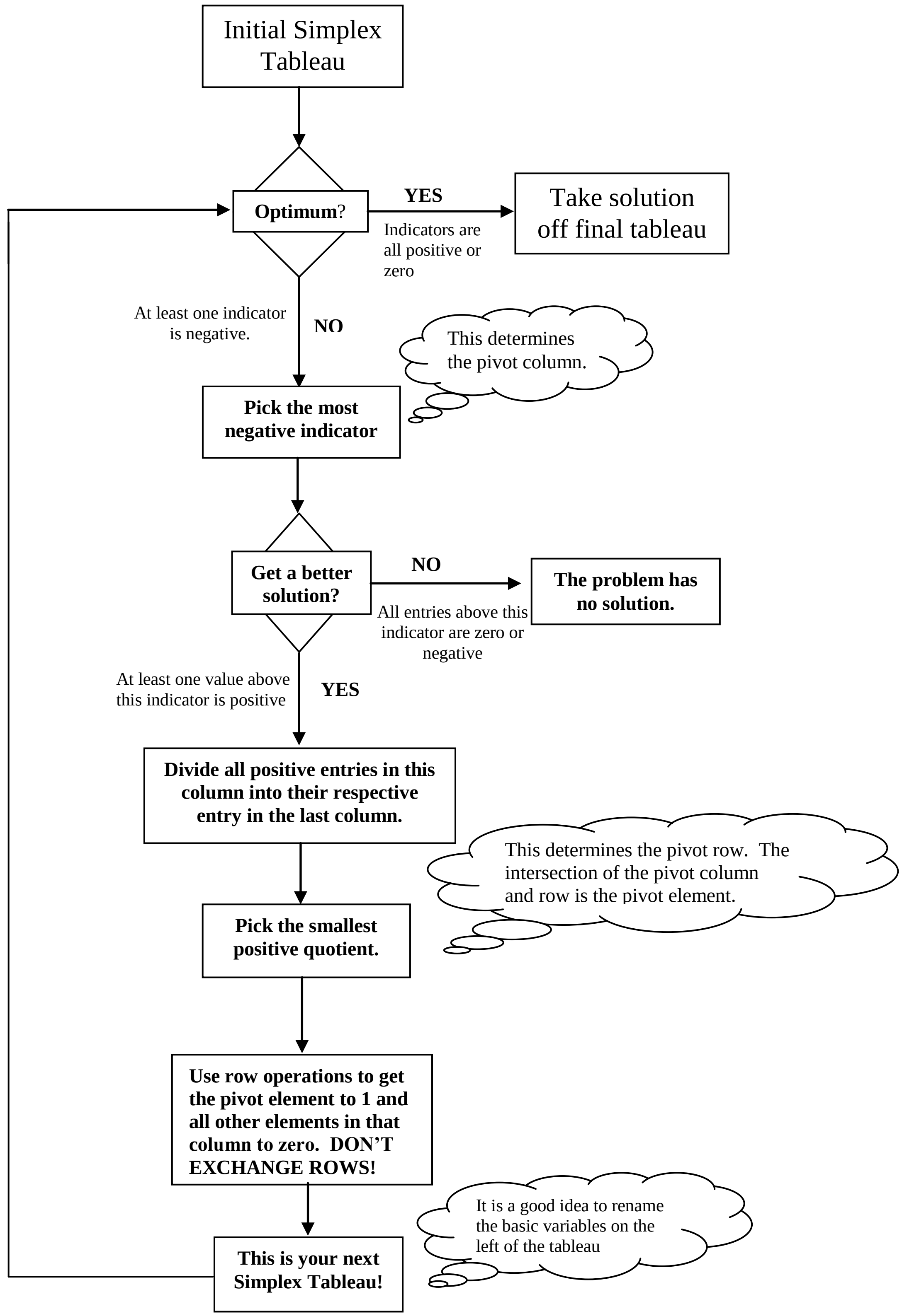
Initial system: 3 basic and 2 non-basic variables	$x_1 + 2x_2 + s_1 = 10$ $3x_1 + 2x_2 + s_2 = 18$ $-20x_1 - 30x_2 + P = 0$ $x_1, x_2, s_1, s_2 \ge 0$	Decision variables: x_1, x_2 Slack variables: s_1, s_2 Constraints (= b where b≥0) Non-zero constraints (≥0)
Fundamental Theorem	If an optimum occurs, it will occur at one (or more) of the basic feasible solutions. These are the corner points of the original feasible region.	

Initial Simplex Tableau

$$
\begin{array}{c}
\overbrace{\quad \text{non}-\text{basic} \quad} \quad \overbrace{\quad \text{basic} \quad} \\
\end{array}
$$

$$
\text{Basic variables} \Rightarrow \begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array}
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & P \end{array} \\
\left[\begin{array}{ccccc|c}
1 & 2 & 1 & 0 & 0 & 10 \\
3 & 2 & 0 & 1 & 0 & 18 \\
\hdashline
-20 & -30 & 0 & 0 & 1 & 0
\end{array}\right] \\
\underbrace{\qquad\qquad\qquad}_{\text{indicators in last row}}
\end{array}
$$

Initial basic feasible solution: $x_1 = 0, x_2 = 0$, P=0 ($s_1 = 10, s_2 = 18$)


Initial Simplex Tableau
Optimum?
YES
Indicators are all positive or zero
Take solution off final tableau
At least one indicator is negative.
NO
This determines the pivot column.
Pick the most negative indicator
Get a better solution?
NO
All entries above this indicator are zero or negative
The problem has no solution.
At least one value above this indicator is positive
YES
Divide all positive entries in this column into their respective entry in the last column.
This determines the pivot row. The intersection of the pivot column and row is the pivot element.
Pick the smallest positive quotient.
Use row operations to get the pivot element to 1 and all other elements in that column to zero. DON'T EXCHANGE ROWS!
It is a good idea to rename the basic variables on the left of the tableau
This is your next Simplex Tableau!

Initial simplex tableau with basic variables s_1, s_2, P and nonbasic variables x_1, x_2. Initial basic feasible solution: $x_1 = 0, x_2 = 0$, P=0 ($s_1 = 10, s_2 = 18$)	$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{ccccc\|c} x_1 & x_2 & s_1 & s_2 & P & \\ 1 & 2 & 1 & 0 & 0 & 10 \\ 3 & 2 & 0 & 1 & 0 & 18 \\ \hline -20 & -30 & 0 & 0 & 1 & 0 \end{array}$
Pivot column is x_2 column (indicator = -30). Entering basic variable is x_2 Pivot row is s_1 row (smallest positive quotient is 5) Exiting basic variable is s_1 Pivot element is 2	$\begin{array}{c} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{ccccc\|c} x_1 & x_2 & s_1 & s_2 & P & \\ 1 & 2 & 1 & 0 & 0 & 10 \\ 3 & 2 & 0 & 1 & 0 & 18 \\ \hline -20 & -30 & 0 & 0 & 1 & 0 \end{array}$
Basic feasible solution: $x_1 = 0$, $x_2 = 5$, P = 150 ($s_1 = 0, s_2 = 8$)	$\begin{array}{c} \\ x_2 \\ s_2 \\ P \end{array} \begin{array}{ccccc\|c} x_1 & x_2 & s_1 & s_2 & P & \\ .5 & 1 & .5 & 0 & 0 & 5 \\ 2 & 0 & -1 & 1 & 0 & 8 \\ \hline -5 & 0 & 15 & 0 & 1 & 150 \end{array}$
Pivot column is x_1 column (indicator = -5). Entering basic variable is x_1 Pivot row is s_2 row (smallest positive quotient is 4) Exiting basic variable is s_2 Pivot element is 2	$\begin{array}{c} \\ x_2 \\ s_2 \\ P \end{array} \begin{array}{ccccc\|c} x_1 & x_2 & s_1 & s_2 & P & \\ .5 & 1 & .5 & 0 & 0 & 5 \\ 2 & 0 & -1 & 1 & 0 & 8 \\ \hline -5 & 0 & 15 & 0 & 1 & 150 \end{array}$
All indicators are positive or zero – STOP Basic feasible solution: $x_1 = 4$, $x_2 = 3$, max. P = 170 ($s_1 = 0, s_2 = 0$)	$\begin{array}{c} \\ x_2 \\ x_1 \\ P \end{array} \begin{array}{ccccc\|c} x_1 & x_2 & s_1 & s_2 & P & \\ 0 & 1 & .75 & -.25 & 0 & 3 \\ 1 & 0 & -.5 & .5 & 0 & 4 \\ \hline 0 & 0 & 12.5 & 2.5 & 1 & 170 \end{array}$