

Factoring the Illegal Way

Remember that the standard form of a quadratic expression is $ax^2 + bx + c$.

Example: $6x^2 - 7x - 3$

This is difficult to factor because the coefficient of the squared term (a) is not 1. Therefore I remove the 6 by multiplying it with the c term (-3). My new trinomial is

$$x^2 - 7x - 18$$

Now this trinomial is easily factored into $(x - 9)(x + 2)$.

I did an “illegal move,” and I now need to “undo” it. Since I multiplied by 6 in the first step, to “undo” it, I now divide each constant by 6.

$$\left(x - \frac{9}{6}\right)\left(x + \frac{2}{6}\right)$$

I now have a factored form with fractions. That is not acceptable so I first reduce the fractions to lowest terms.

$$\left(x - \frac{3}{2}\right)\left(x + \frac{1}{3}\right)$$

The binomials still have fractions that cannot be reduced, so I simply take the denominator of the fraction and squeeze it in front of the x in the binomial, making the denominator the coefficient of x.

$$\begin{array}{c} \left(x - \frac{3}{2}\right)\left(x + \frac{1}{3}\right) \\ \text{↖ ↗} \\ (2x - 3)(3x + 1) \end{array}$$

It works every time!

The only problem I have had with students using this method is they forget to undo the illegal move. You must do both steps!!!

This does not work if a is a negative number. Factor out a -1 and then proceed.

Here's another example:

$$2x^2 - 7x - 15$$

Because a is not 1, I perform the illegal move. **Multiply by 2.**

$$x^2 - 7x - 30$$

Factor using the sign clues.

$$(x - 10)(x + 3)$$

Now I must undo the illegal move. **Divide by 2.**

$$\left(x - \frac{10}{2}\right)\left(x + \frac{3}{2}\right)$$

Simplify the fractions if I can.

$$(x - 5)\left(x + \frac{3}{2}\right)$$

Since I still have a fraction, I move the denominator of the fraction in front of the variable.

$$(x - 5)(2x + 3)$$

One More Example:

$$12x^2 + 17x + 6$$

Perform the illegal move. **Multiply by 12.**

$$x^2 + 17x + 72$$

Factor using the sign clues.

$$(x + 8)(x + 9)$$

Undo the illegal move. **Divide by 12.**

$$\left(x + \frac{8}{12}\right)\left(x + \frac{9}{12}\right)$$

Reduce fractions.

$$\left(x + \frac{2}{3}\right)\left(x + \frac{3}{4}\right)$$

Remove fractions.

$$(3x + 2)(4x + 3)$$