Objectives

1. Add and subtract functions
2. Multiply functions **Skip objective 3 – Divide functions.**
3. Find the composition of two functions.

Vocabulary

Composition of functions	$(f \circ g)(x) = f[g(x)]$ for all x in the domain of g for which g(x) is in the domain of f.

Prior Knowledge

1. f(x) and y are interchangeable.

2. Function notation f(x) says the relation is a function and the variable is x.

3. Given a function f(x), f(5) says to evaluate the function when x = 5.

New Concepts: **Skip objective 3 – Divide functions.**

1. **Add or subtract functions.**
2. **Multiply functions**

Functions can be added, subtracted or multiplied.

$(f + g)(x) = f(x) + g(x)$
$(f – g)(x) = f(x) – g(x)$
$(fg)(x) = f(x)\cdot g(x)$

Example 1: f(x) = 5x + 1 g(x) = x – 8

(f + g)(x)	(f – g)(x)	(fg)(x)
f(x) + g(x) (5x + 1) + (x – 8) 6x – 7	f(x) – g(x) (5x + 1) – (x – 8) 5x + 1 – x + 8 4x + 9	f(x) ·g(x) (5x + 1)(x – 8) $5x^2 – 40x + 1x – 8$ $5x^2 – 39x – 8$

Example 2: $f(x) = -x^2 – 8x + 2$ $g(x) = x^2 + 2$

(f + g)(x)	(f – g)(x)	(fg)(x)
f(x) + g(x) $(-x^2 – 8x + 2) + (x^2 + 2)$ -8x + 4	f(x) – g(x) $(-x^2 – 8x + 2) - (x^2 + 2)$ $-x^2 – 8x + 2 – x^2 – 2$ $-2x^2 – 8x$	f(x) ·g(x) $(-x^2 – 8x + 2)(x^2 + 2)$ $-x^4 -2x^2 -8x^3 -16x + 2x^2 + 4$ $-x^4 -8x^3 – 16x + 4$

Example 3: f(x) = 2x – 5 g(x) = 3x – 7

(f + g)(x)	(f – g)(x)	(fg)(x)
f(x) + g(x) (2x – 5) + (3x – 7) 5x – 12	f(x) – g(x) (2x – 5) – (3x – 7) 2x – 5 – 3x + 7 -x + 2	f(x) ·g(x) (2x – 5)(3x – 7) $6x^2 – 29x + 35$

Example 4: Use Example 1 to find each of the following.

(f + g)(-2)	(f – g)(5)	(fg)(-1)
(f + g)(x) = 6x – 7 (f + g)(-2) = 6(-2) – 7 (f + g)(-2) = -12 – 7 (f + g)(-2) =-19	(f – g)(x) = 4x + 9 (f – g)(5) = 4(5) + 9 (f – g)(5) = 20 + 9 (f – g)(5) = 29	(fg)(x) = $5x^2$ – 39x – 8 (fg)(-1) = $5(-1)^2$ – 39(-1) – 8 (fg)(-1) = 5 + 39 – 8 (fg)(-1) = 36

12.1.1 Composition of functions.

Composition of functions is shorthand notation for substitution.

$$(f \circ g)(x) = f(g(x))$$

This says to find the value of g(x) and then substitute that value for x in f(x). It's a two-step problem.

Example 5: If f(x) = 3x + 5 and g(x) = x^2 + 3, find $(f \circ g)(2)$.

$(f \circ g)(2) = f(g(2))$	Rewrite in nested format.
g(2) = $(2)^2$ + 3 = 7	Work from the inside out. Substitute 2 for x in g(x) and evaluate.
f(7) = 3(7) + 5 = 26	Substitute the value from the first part for x in the f function.

Example 6: If f(x) = 3x + 5 and g(x) = x^2 + 3, find $(g \circ f)(2)$.

$(g \circ f)(4) = g(f(2))$	Rewrite in nested format.
f(2) = 3(2) + 5 = 11	Work from the inside out.
g(11) = $(11)^2$ + 3 = 124	Substitute the value from the first part for x in the second function.

Example 7: If f(x) = 3x + 5 and g(x) = x^2 + 3, find $(f \circ g)(a)$.

$(f \circ g)(2) = f(g(a))$	Rewrite in nested format.
g(a) = a^2 + 3	Work from the inside out. Substitute a for x in g(x),
f(a^2 + 3) = 3(a^2 + 3) + 5	Substitute the value from the first part for x in the f function.
=3a^2 + 9 + 5	
=3a^2 + 14	

Example 8: If f(x) = 3x + 5 and g(x) = x^2 + 3, find $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x))$	Rewrite in nested format.
g(3x + 5)	Since there is no number to substitute for x, replace f(x) with the expression.
g(3x + 5) = $(3x + 5)^2$ + 3	Substitute and simplify.
= $9x^2$ + 30x + 25 + 3	Remember to rewrite and foil.
= $9x^2$ + 30x + 28	