1. Let $G$ be a group and recall that the non-negative powers of an element $x$ in $G$ are defined as follows: $x^0 = e$, $x^1 = x$, and $x^{n+1} = x^n x$ for $n > 0$. Now, let $x$ and $y$ be elements in $G$ and, using induction, show that $(x y x^{-1})^n = x y^n x^{-1}$ holds for $n > 0$.

2. (a) What property tells us the set $S = \{x \in \mathbb{Z}^+ | x = 280s + 595t$ for some $s \& t \in \mathbb{Z}\}$ has a least member?

(b) Find the least member of the set $S$ in part (a):

3. Which congruence classes in $\mathbb{Z}_9$ form a group under multiplication mod 9?
   The following classes:

   Is this group cyclic? If it is, then identify a generator:

4. Solve the following system in $\mathbb{Z}_7$: $[2][x] + [3][y] = [5] \& [4][x] + [2][y] = [4]$
5. Let $\phi : G \to G^*$, a mapping from the group $G$ into another group $G^*$, be a homomorphism. Let $e$ be the identity in $G$ & $e^*$ the identity in $G^*$. Consider the following statements:

1. $\phi$ takes the identity of $G$ to the identity of $G^*$, i.e. $\phi (e) = e^*$.
2. For $x \in G$, $\phi (x^{-1}) = (\phi (x))^{-1}$, i.e. $\phi$ maps inverses to inverses.
3. The pre-image of a normal subgroup in $G^*$ is normal in $G$.
4. The image of a normal subgroup in $G$ is normal in $G^*$.
5. The kernel of $\phi$ is a normal subgroup of $G$.

Then

(a) statement 4 is false, but the others are all true.
(b) statements 1 & 2 are true, but not 3, 4, & 5.
(c) each of these statements is true.
(d) only statement 3 is false.

6. In proving the uniqueness of the group identity, which of the following properties is needed?

(a) the commutative property
(b) the associative property
(c) none of these

7. In showing the uniqueness of inverses in a group, which of the following properties is needed?

(a) the commutative property
(b) the associative property
(c) none of these
8. Consider the relation $R$ defined on the set $\mathbb{Z}$ of integers:

$xRy$ if and only if 7 divides $8x - 15y$

(a) Show that $R$ is an equivalence relation:

$R$ is reflexive:

$R$ is symmetric:

$R$ is transitive:

(b) Describe the distinct equivalence classes of $R$:

9. A subset $S$ of the ring $R$ is a subring of $R$ iff the following conditions hold:

(a) $S$ is nonempty,
(b) $x \in S$ implies that $-x \in S$
(c) $x$ and $y \in S$ imply that $x + y$ & $xy \in S$

True False

10. In a multiplicative group $G$, given that we have defined $x^n$ for $x \in G$ and $n$ a positive integer, how should we define $x^k$ if $k$ is a negative integer?

11. If every proper subgroup of a group $G$ is abelian, then $G$ itself is abelian.

True False

12. In a group $G$, is the normality of subgroups a transitive property?

Yes No
13. Show the mapping $\phi$ defined by $\phi(a + bi) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a homomorphism from the multiplicative group of non-zero complex numbers $\mathbb{C}\setminus\{0\}$ into the multiplicative group of real invertible $2 \times 2$ matrices $H = \{A \in M_2(\mathbb{R}) \mid \det(A) > 0\}$.

Fill in the blanks:

14. Assume that $\phi$ is an monomorphism from the group $G$ with identity $e$ into the group $G^*$ with identity $e^*$. Then we know that $\ker \phi = \ldots$.

15. $\mathbb{Z}_n$ is a field if and only if $n$ is \ldots.