NAME________________________________ SCORE______________

EXERCISE 4
EARTH SUN RELATIONSHIPS

ORBIT

The earth revolves the sun in an elliptical orbit. This implies that the earth, during its revolution is at some point at a minimum distance to the sun (91.5 million miles) and at some point a maximum distance from the sun (94.5 million miles). Figure 4.1 shows this relationship.

The sun provides the energy that drives earth's systems. The sun's energy travels to earth as an electromagnetic spectrum. The reception of this energy on earth is referred to as insolation (incoming solar radiation). The amount of solar energy that is received by the earth from the sun is but a small amount of the sun's total energy output. The amount of solar energy received by the earth is constant at 2 cal/ cm^2 / min^{-1} and is called the solar constant. The sun's energy travels to the earth as parallel rays. The earth's surface is spherical therefore the sun's rays will be vertical to an observer (90° to the surface) at only one place. The meridian of the observer would experience solar noon at all places on the meridian. The latitude of the observer standing on the solar noon meridian is called the declination latitude of the observer or the zenith position of the sun. Figure 4.2 shows this relationship.

The earth's axis is tilted at an angle of 66 1/ 2 degrees to the plane of its orbit around the sun. As the earth revolves around the sun, the axis remains in a parallel position. This phenomena results in an apparent motion of the sun moving from a zenith position of 23.5 degrees north latitude (tropic of cancer) on June 22 (summer solstice) to 23.5 degrees south latitude (tropic of capricorn) on December 22 (winter solstice). While moving to these two solstice positions, the sun is at zenith on the equator on March 22 the spring equinox and on September 22 the fall equinox. This relationship between the earth and the sun is shown in figure 4.3

The earth rotates around its axis in a west to east direction. This rotation direction results in the sun rising in the east and setting in the west while reaching its maximum height at solar noon for any place on earth. On March 22 the sun's zenith position would be directly overhead on the equator, while at the north or south poles the sun's rays would be tangental to the earth's surface and the sun would appear on the horizon. If you were standing on the equator on march 22 the sun would be 90° overhead. If you were in Billings, Montana (45° 47′ N, 108° 32′ W) on march 22 the noon sun angle would be 45° above the southern horizon. (maximum latitude of 90°= perpendicular, minus the latitude of 45° =45°).

The sun angle decreases by 1° for every degree of latitude between a person's position and the location where the sun's rays are vertical. Remember that the sun has an apparent shift from 23.5° N to 23.5° S during the year. The zenith position of the sun also changes between 23.5° N and 23.5° S. This zenith position of the sun is called the declination angle of the sun or simply the sun's declination.

The change in the sun's declination angle changes the angle that an observer would observe the sun above their horizon. For example, an observer located in Billings, Montana on December 22 would observe the noon sun angle at 20.8° above their horizon.

The formula used before now must change to include the sun's declination angle. Therefore the following equation must be used to calculate the sun angle above the horizon:

$$SA\text{ (sun angle)} = 90^\circ - \text{latitude (of observer)} \pm \text{sun declination}$$

For Billings, Montana on December 22, the sun declination is 23.5° S therefore:

$$SA = 90^\circ - 45^\circ \pm 23.5^\circ$$

$$SA = 21.5^\circ$$

The latitude of an observer can be determined by reworking the sun angle equation as shown below:

$$\text{Latitude} = 90^\circ - SA \pm \text{declination}$$

For Billings, Montana on December 22:

$$\text{latitude} = 90^\circ - 21.5 \pm 23.5^\circ$$

$$\text{latitude} = 45^\circ$$

1. Calculate the noon sun angle for Cairo, Illinois (37° N), on the following dates:

   a) June 22 ______________________________

   b) December 22__________________________

   c) March and September 22 _________________

2. Calculate the latitude of a location where the noon sun angle is 45° above the northern horizon on December 22.

   ____

3. Calculate the latitude of a location where the noon sun angle is 15° above the southern horizon on December 22.

   ____

4. Calculate the noon sun angle for Billings, Montana on the following dates:

   a) June 22 _____________________

   b) December 22 ________________

The Analemma

The zenith (90° overhead) position of the sun varies during the year from 23.5° N (the tropic of Cancer) on June 22, to 23.5° S (the tropic of Capricorn) on December 22. Between these two dates and latitudes, the sun's zenith position and angle changes for any day of the year. The rate at which the sun angle changes with latitude is not constant . The analemma (figure 4.4) is a graph that shows the latitude at which the sun's rays are at zenith on any day of the year.

Example use of the analemma:

Calculate the noon sun angle for Boulder, Colorado 40° N latitude on May 5.

Ans. From the analemma we find that the zenith position of the sun is 16° N latitude.

using the equation $SA = 90^\circ - \text{latitude} \pm \text{declination}$

$SA = 90^\circ - 40^\circ \pm 16^\circ$

$SA = 66^\circ$

5. Using the analemma, calculate the noon sun angle for Billings, Montana on the following dates:

June 22 ____________

July 1 ______________

August 1 ___________

September 30 _______

October 1 ____________

October 15 ___________

November 1 __________

December 31 __________

6. Between what dates do the zenith rays of the sun appear to change declination most rapidly?______________________ most slowly? ______________________

Solar Intensity

Remember that the earth's surface is curved, therefor the sun's rays striking the earth's surface at any position other than at the zenith position will be at some angle less than 90°. This smaller angle results in less intense radiation per unit area. The intensity of the radiation varies with the sine of the angle at which the rays are striking the earth's surface. The earth receives short wave radiation (ultraviolet) from the sun and reradiates long wave (infrared) the efficiency of radiation varies with changes in latitude. The percent efficiency of solar radiation for any latitude on any day can be calculated by using the following equation.

% efficiency = sun angle x sine of the sun angle

Example problem:

Calculate the percent efficiency of solar radiation for Boulder, Colorado (40 N lat.) on June 22.

a) from the analemma one finds the zenith position of the sun to be 23.5 N latitude

b) Using the equation for calculating the sun angle one finds the sun angle for Boulder on June 22.

$$SA = 90^\circ - 40^\circ \pm 23.5^\circ$$

$$SA = 73.5^\circ$$

sine of the sun angle = 0.958

$73.5^\circ \times 0.958 = 70.47$ % efficiency

7. Calculate the percent efficiency of solar radiation for the following locations and dates:

	June 22	December 22
a) Barrow, Alaska 71° N	__________	__________
b) Little Rock, Arkansas 34° N	__________	__________
c) Mexico City, Mexico 19° N	__________	__________
d) Nairobi, Kenya 1° S	__________	__________
e) Melbourne, Australia 37° S	__________	__________
f) Punta Arenas, Chile 53° S	__________	__________

Figure 4.1 Earth's orbit

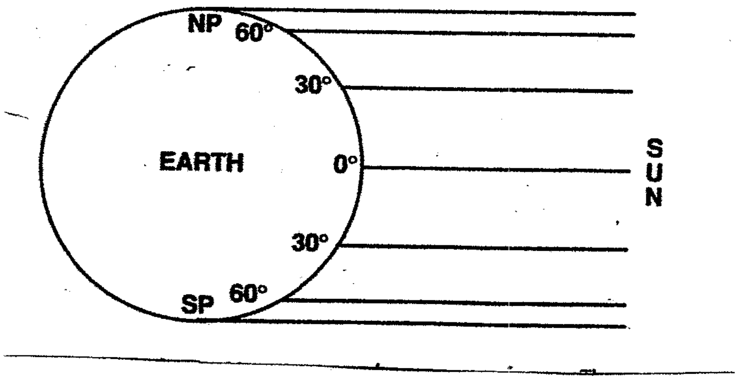

Figure 4.2 Solar Ray Angles on an Equinox

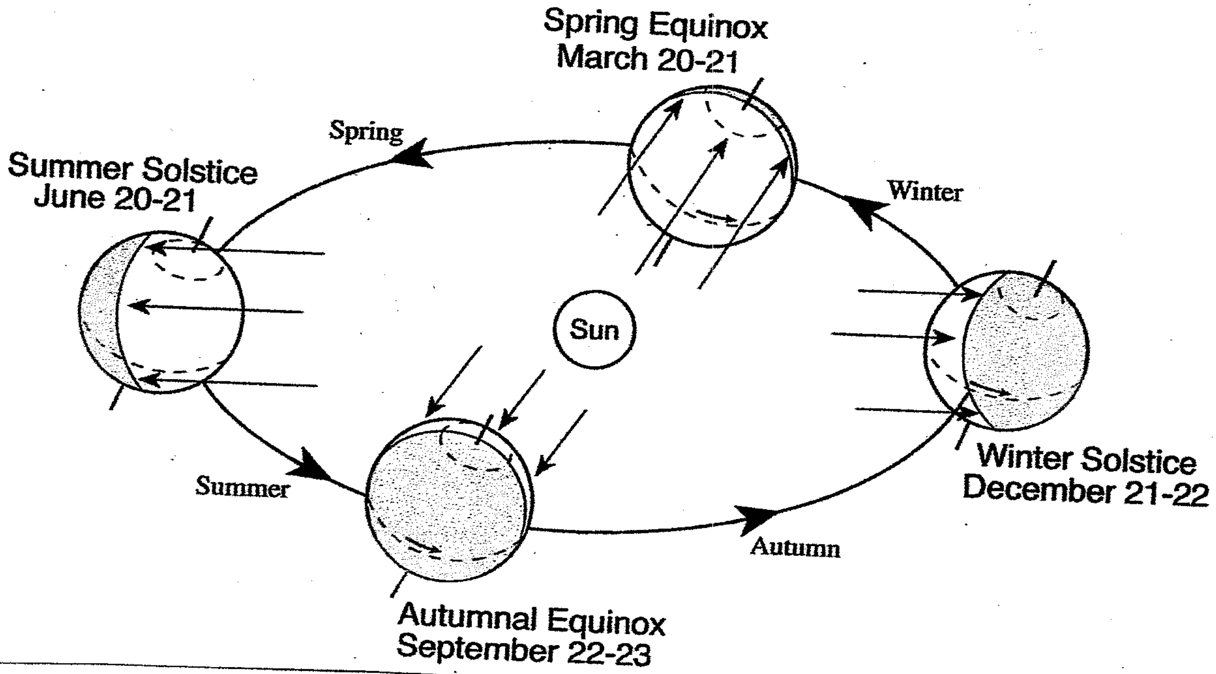

Figure 4.3 Earth - Sun Relationship

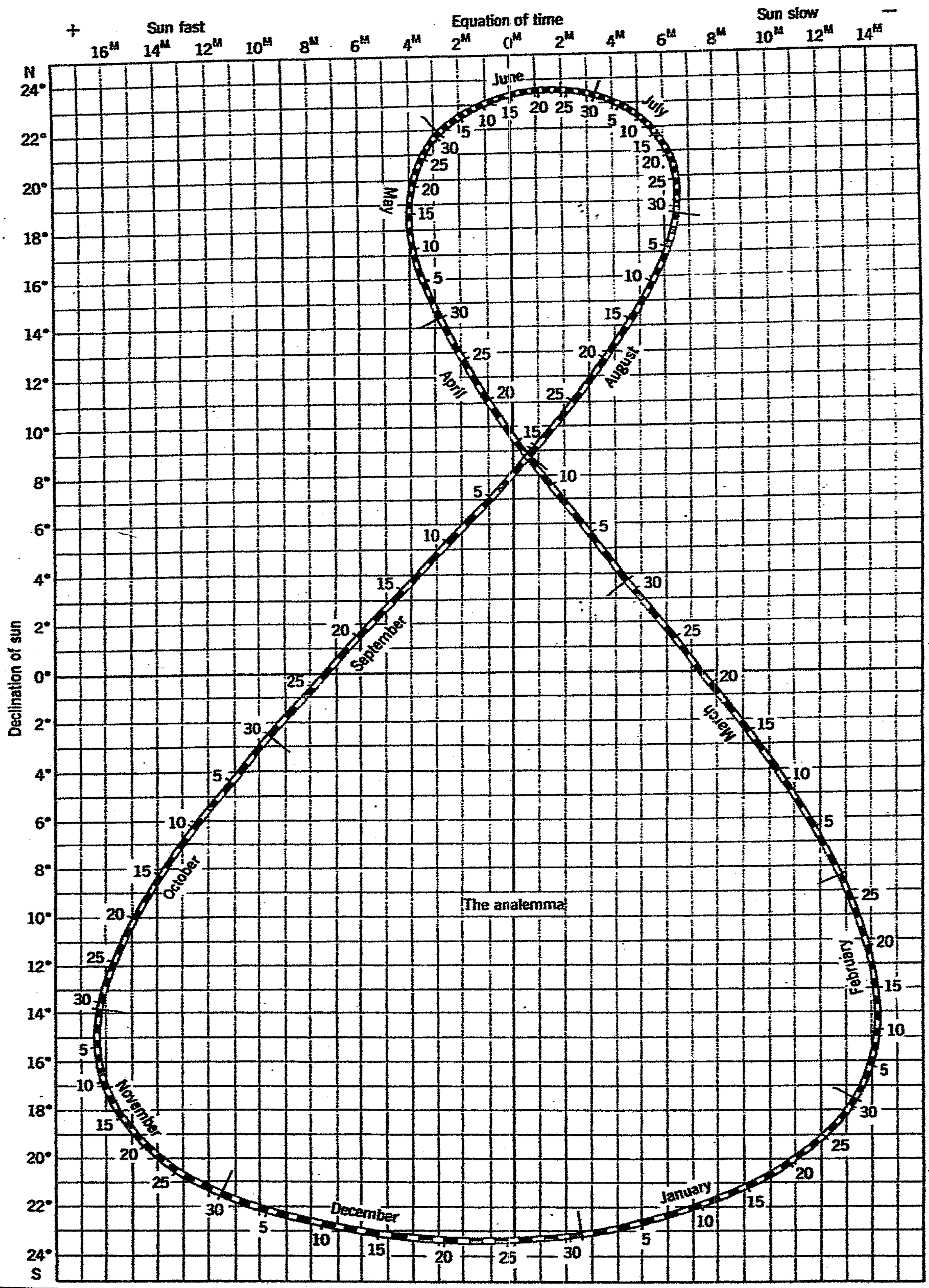

Figure 4.4 The ANALEMA gives the declination of the SUN AND the equation of time FOR EACH day in the year