# Continuous Random Variables

Chapter 5

5.2 The uniform distribution is sometimes referred to as the **randomness** distribution.

5.4 From Exercise 5.3, $f(x) = 1/20 = .05$ $(10 \le x \le 30)$

a. $P(10 \le x \le 25) = (25 - 10)(.05) = .75$

b. $P(20 < x < 30) = (30 - 20)(.05) = .5$

c. $P(x \ge 25) = (30 - 25)(.05) = .25$

d. $P(x \le 10) = (10 - 10)(.05) = 0$

e. $P(x \le 25) = (25 - 10)(.05) = .75$

f. $P(20.5 \le x \le 25.5) = (25.5 - 20.5)(.05) = .25$

5.6 From Exercise 5.5, $f(x) = \frac{1}{2}$ $(2 \le x \le 4)$

a. $P(x \ge a) = .5 \Rightarrow (4 - a)\left(\frac{1}{2}\right) = .5$

$\Rightarrow \quad 4 - a = 1$

$\Rightarrow \quad a = 3$

b. $P(x \le a) = .2 \Rightarrow (a - 2)\left(\frac{1}{2}\right) = .2$

$\Rightarrow \quad a - 2 = .4$

$\Rightarrow \quad a = 2.4$

c. $P(x \le a) = 0 \Rightarrow (a - 2)\left(\frac{1}{2}\right) = 0$

$\Rightarrow \quad a - 2 = 0$

$\Rightarrow \quad a = 2$ or any number less than 2

d. $P(2.5 \le x \le a) = .5 \Rightarrow (a - 2.5)\left(\frac{1}{2}\right) = .5$

$\Rightarrow \quad a - 2.5 = 1$

$\Rightarrow \quad a = 3.5$

5.8 $\mu = \frac{c+d}{2} = 50 \Rightarrow c+d = 100 \Rightarrow c = 100 - d$

$$\sigma = \frac{d-c}{\sqrt{12}} = 5 \Rightarrow d - c = 5\sqrt{12}$$

Substituting, $d - (100 - d) = 5\sqrt{12} \Rightarrow 2d - 100 = 5\sqrt{12}$

$$\Rightarrow 2d = 100 + 5\sqrt{12}$$

$$\Rightarrow d = \frac{100 + 5\sqrt{12}}{2}$$

$$\Rightarrow d = 58.66$$

Since $c + d = 100 \Rightarrow c + 58.66 = 100$

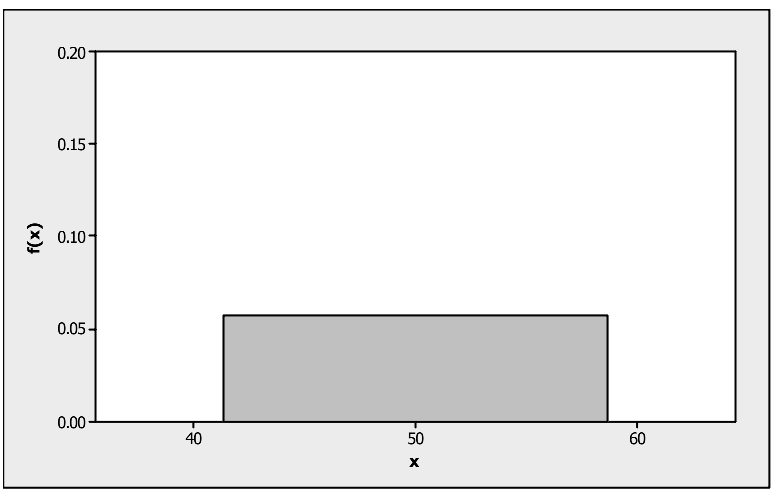
$$\Rightarrow c = 41.34$$

$f(x) =$ $f(x) = \frac{1}{d-c} \quad (c \le x \le d)$

$$\frac{1}{d-c} = \frac{1}{58.66 - 41.34} = \frac{1}{17.32} = .058$$

Therefore, $f(x) = \begin{cases} .058 & 41.34 \le x \le 58.66 \\ 0 & \text{otherwise} \end{cases}$

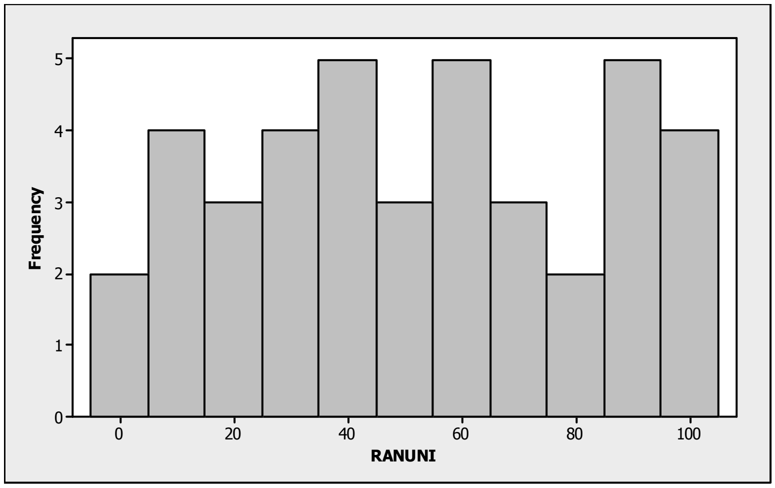
The graph of the probability distribution for *x* is given here.




5.10 We are given that *x* is a uniform random variable on the interval from 0 to 3,600 seconds. Therefore, $c = 0$ and $d = 3{,}600$. The last 15 minutes would represent 15(60) = 900 seconds and would represents seconds 2700 to 3600.

$$P(x > 2700) = \frac{3600 - 2700}{3600 - 0} = \frac{900}{3600} = .25$$

5.12 Using MINITAB, the histogram of the data is:




The histogram looks like the data could come from a uniform distribution. There is some variation in the height of the bars, but we cannot expect a perfect graph from a sample of only 40 observations.

5.14 a. Since $x$ is a uniform random variable on the interval from 0 to 1, $c = 0$ and $d = 1$.

$$E(x) = \frac{c+d}{2} = \frac{0+1}{2} = \frac{1}{2} = .5$$

b. $$P(x > .7) = \frac{1-.7}{1-0} = \frac{.3}{1} = .3$$

c. If there are only 2 members, then there is only 1 possible connection. The density will then be either 0 or 1. The density would be a discrete random variable, not continuous. Therefore, the uniform distribution would not be appropriate.

5.16 Let $x$ = length of time a bus is late. Then $x$ is a uniform random variable with probability distribution:

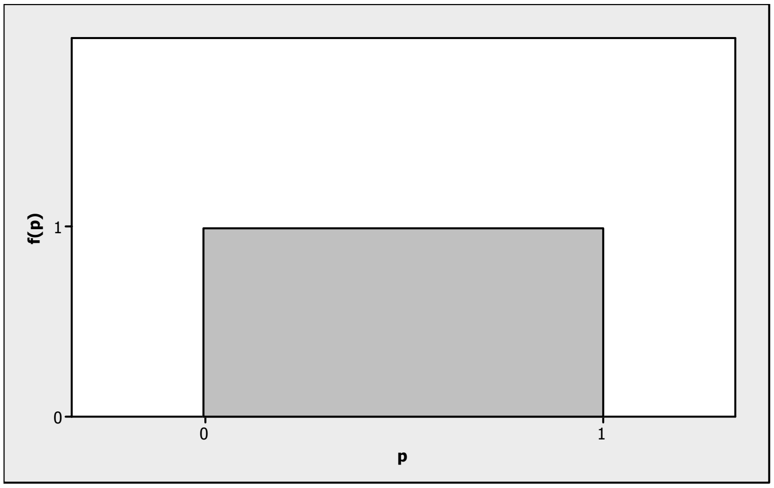
$$f(x) = \begin{cases} \frac{1}{20} & (0 \le x \le 20) \\ 0 & \text{otherwise} \end{cases}$$

a. $$\mu = \frac{0+20}{2} = 10$$

b. $$P(x \ge 19) = (20-19)\left(\frac{1}{20}\right) = \frac{1}{20} = .05$$

c. It would be doubtful that the director's claim is true, since the probability of the being more than 19 minutes late is so small.

5.18 a.




b. $\mu = \frac{c+d}{2} = \frac{0+1}{2} = .5$

$\sigma = \frac{d-c}{\sqrt{12}} = \frac{1-0}{\sqrt{12}} = .289$ $\quad\quad \sigma^2 = .289^2 = .083$

c. $P(p > .95) = (1 - .95)(1) = .05$

$P(p < .95) = (.95 - 0)(1) = .95$

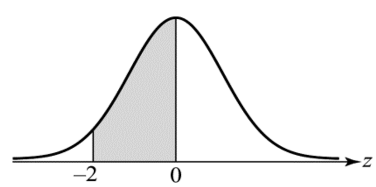
d. The analyst should use a uniform probability distribution with *c* = .90 and *d* = .95.

$$f(p) = \begin{cases} \frac{1}{d-c} = \frac{1}{.95-.90} = \frac{1}{.05} = 20 & (.90 \le p \le .95) \\ 0 & \text{otherwise} \end{cases}$$

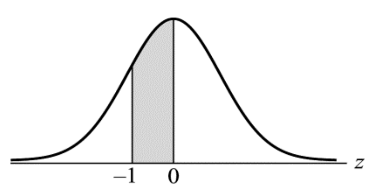
5.20 A normal distribution is a bell-shaped curve with the center of the distribution at $\mu$.

5.22 If *x* has a normal distribution with mean $\mu$ and standard deviation $\sigma$, then the distribution of $z = \frac{x-\mu}{\sigma}$ is a normal with a mean of $\mu = 0$ and a standard deviation of $\sigma = 1$.

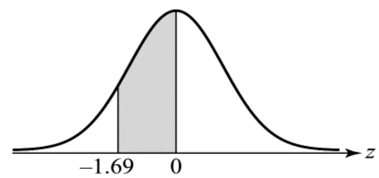
5.24 a. $P(-2.00 < z < 0) = .4772$
(from Table IV, Appendix A)



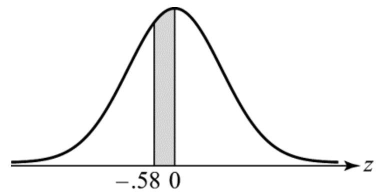

b. $P(-1.00 < z < 0) = .3413$
(from Table IV, Appendix A)



c. $P(-1.69 < z < 0) = .4545$
(from Table IV, Appendix A)

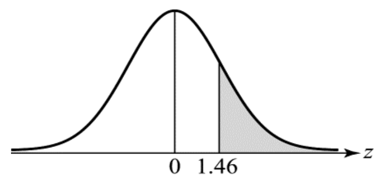



d. $P(-.58 < z < 0) = .2190$
(from Table IV, Appendix A)

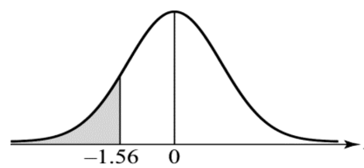



5.26 Using Table IV, Appendix A:

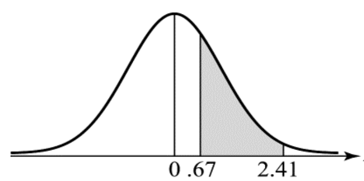
a. $P(z > 1.46) = .5 - P(0 < z \le 1.46)$
$= .5 - .4279 = .0721$



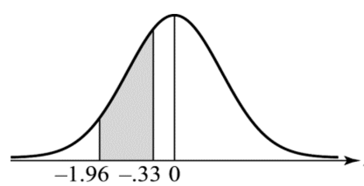

b. $P(z < -1.56) = .5 - P(-1.56 \le z < 0)$
$= .5 - .4406 = .0594$



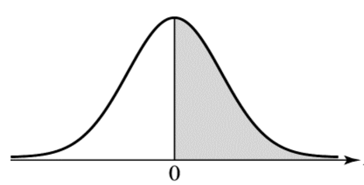

c. $P(.67 \le z \le 2.41)$
$= P(0 < z \le 2.41) - P(0 < z < .67)$
$= .4920 - .2486 = .2434$



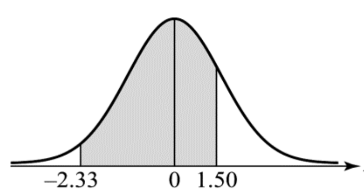

d. $P(-1.96 \le z < -.33)$
$= P(-1.96 \le z < 0) - P(-.33 \le z < 0)$
$= .4750 - .1293 = .3457$



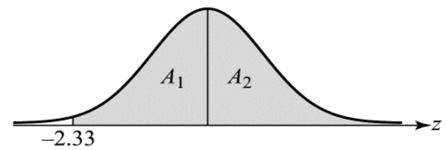

e. $P(z \ge 0) = .5$



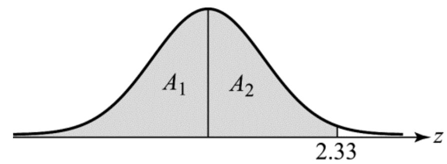

f. $P(-2.33 < z < 1.50)$
$= P(-2.33 < z < 0) + P(0 < z < 1.50)$
$= .4901 + .4332 = .9233$



g. $P(z \geq -2.33) = P(-2.33 \leq z \leq 0) + P(z \geq 0)$
$= .4901 + .5000$
$= .9901$



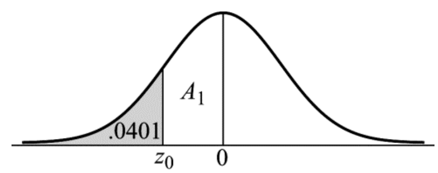

h. $P(z < 2.33) = P(z \leq 0) + P(0 \leq z \leq 2.33)$
$= .5000 + .4901$
$= .9901$




5.28 Using Table IV of Appendix A:

a. $P(z < z_o) = .0401$

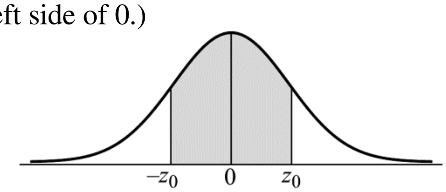
$A_1 = .5000 - .0401 = .4591$




Look up the area .4591 in the body of Table IV;
$z_0 = -1.75$

($z_0$ is negative since the graph shows $z_0$ is on the left side of 0.)

b. $P(-z_o \leq z \leq z_o) = .95$

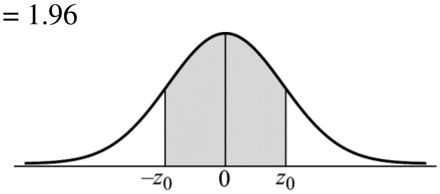



$P(-z_o \leq z \leq z_o) = 2P(0 \leq z \leq z_o)$
$2P(0 \leq z \leq z_o) = .95$

Therefore, $P(0 \leq z \leq z_o) = .4750$

Look up the area .4750 in the body of Table IV; $z_0 = 1.96$

c. $P(-z_o \leq z \leq z_o) = .90$

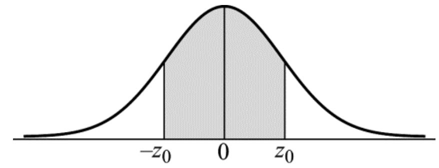



$P(-z_o \leq z \leq z_o) = 2P(0 \leq z \leq z_o)$
$2P(0 \leq z \leq z_o) = .90$

Therefore, $P(0 \leq z \leq z_o) = .45$

Look up the area .45 in the body of Table IV; $z_0 = 1.645$ (.45 is half way between .4495 and .4505; therefore, we average the $z$-scores $\frac{1.64 + 1.65}{2} = 1.645$)

d. $P(-z_o \le z \le z_o) = .8740$



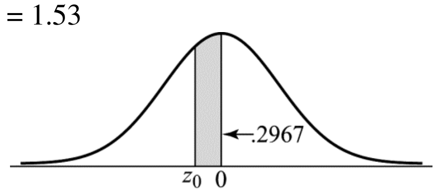

$P(-z_o \le z \le z_o) = 2P(0 \le z \le z_o)$

$2P(0 \le z \le z_o) = .8740$

Therefore, $P(0 \le z \le z_o) = .4370$

Look up the area .4370 in the body of Table IV; $z_0$ = 1.53

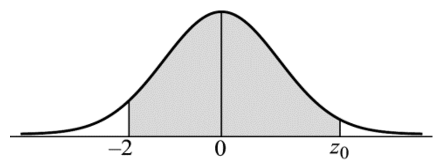
e. $P(-z_o \le z \le 0) = .2967$




$P(-z_o \le z \le 0) = P(0 \le z \le z_o)$

Look up the area .2967 in the body of Table IV; $z_0$ = .83 and -$z_0$ = -.83

f. $P(-2 < z < z_o) = .9710$



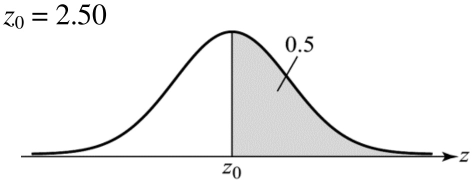

$P(-2 < z < z_o)$
$= P(-2 < z < 0) + P(0 < z < z_0) = .9710$

$P(0 < z < 2) + P(0 < z < z_0) = .9710$

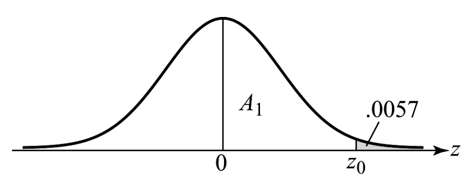
Thus, $P(0 < z < z_0) = .9710 - P(0 < z < 2) = .9710 - .4772 = .4938$

Look up the area .4938 in the body of Table IV; $z_0$ = 2.50

g. $P(z \ge z_0) = .5$
$z_0 = 0$




h. $P(z \ge z_0) = .0057$
$A_1 = .5 - .0057 = .4943$
Looking up the area .4943 in Table IV
gives $z_0$ = 2.53.




5.30 Using the formula $z = \dfrac{x-\mu}{\sigma}$ with $\mu = 25$ and $\sigma = 5$:

a. $z = \dfrac{25-25}{5} = 0$

b. $z = \dfrac{30-25}{5} = 1$

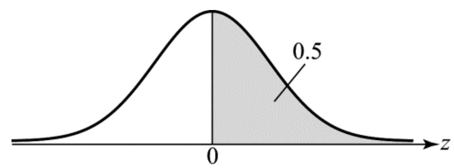
c. $z=\frac{37.5-25}{5}=2.5$

d. $z=\frac{10-25}{5}=-3$

e. $z=\frac{50-25}{5}=5$

f. $z=\frac{32-25}{5}=1.4$

5.32 a. $P(x \ge x_0)=.5 \Rightarrow P\left(z \ge \frac{x_0-30}{8}\right)$

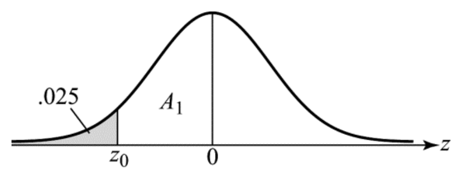



$= P(z \ge z_0) = .5$

$\Rightarrow z_0 = 0 = \frac{x_0-30}{8}$

$\Rightarrow x_0 = 8(0)+30 = 30$

b. $P(x < x_0)=.025 \Rightarrow P\left(z < \frac{x_0-30}{8}\right)$



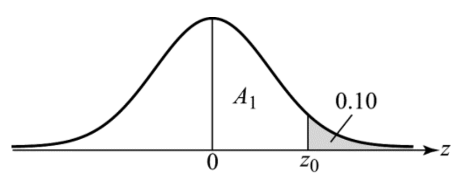

$= P(z < z_0) = .025$

$A_1 = .5-.025 = .4750$

Looking up the area .4750 in Table IV gives $z_0$ = 1.96.
Since $z_0$ is to the left of 0, $z_0 = -1.96$.

$z_0 = -1.96 = \frac{x_0-30}{8} \Rightarrow x_0 = 8(-1.96)+30 = 14.32$

c. $P(x > x_0)=.10 \Rightarrow P\left(z > \frac{x_0-30}{8}\right)$



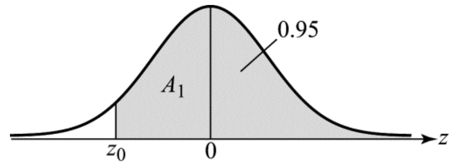

$= P(z > z_0) = .10$

$A_1 = .5-.10 = .4000$

Looking up the area .4000 in Table IV gives $z_0$ = 1.28.

$z_0 = 1.28 = \frac{x_0-30}{8} \Rightarrow x_0 = 8(1.28)+30 = 40.24$

d. $P(x > x_0) = .95 \Rightarrow P\left(z > \frac{x_0 - 30}{8}\right)$



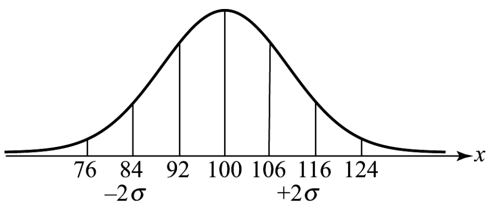

$= P(z > z_0) = .95$

$A_1 = .95 - .50 = .4500$

Looking up the area .4500 in Table IV gives $z_0$ = 1.645.
Since $z_0$ is to the left of 0, $z_0$ = −1.645.

$z_0 = -1.645 = \frac{x_0 - 30}{8} \Rightarrow x_0 = 8(-1.645) + 30 = 16.84$

5.34




Using Table IV, Appendix A:

a. $P(\mu - 2\sigma \le x \le \mu + 2\sigma) = P(-2 \le z \le 2)$
$= P(-2 \le z \le 0) + P(0 \le z \le 2)$
$= .4772 + .4772 = .9544$

b. $P(x \ge \mu + 2\sigma) = P(z \ge 2) = .5 - .4772 = .0228$

c. $P(x \le 92) = P\left(z \le \frac{92 - 100}{8}\right) = P(z \le -1) = .5 - .3413 = .1587$

d. $P(92 \le x \le 116) = P\left(\frac{92 - 100}{8} \le z \le \frac{116 - 100}{8}\right) = P(-1 \le z \le 2)$
$= P(-1 \le z \le 0) + P(0 \le z \le 2)$
$= .3413 + .4772 = .8185$

e. $P(92 \le x \le 96) = P\left(\frac{92 - 100}{8} \le z \le \frac{96 - 100}{8}\right) = P(-1 \le z \le -.5)$
$= P(-1 \le z \le 0) - P(-.5 \le z \le 0)$
$= .3413 - .1915 = .1498$

f. $P(76 \le x \le 124) = P\left(\frac{76 - 100}{8} \le z \le \frac{124 - 100}{8}\right) = P(-3 \le z \le 3)$
$= P(-3 \le z \le 0) + P(0 \le z \le 3)$
$= .4987 + .4987 = .9974$

5.36 a. $P(x < \mu - 2\sigma) + P(x > \mu + 2\sigma) = P(z < -2) + P(z > 2)$
$$= (.5 - .4772) + (.5 - .4772)$$
$$= 2(.5 - .4772) = .0456$$
(from Table IV, Appendix A)

$P(x < \mu - 3\sigma) + P(x > \mu + 3\sigma) = P(z < -3) + P(z > 3)$
$$= (.5 - .4987) + (.5 - .4987)$$
$$= 2(.5 - .4987) = .0026$$
(from Table IV, Appendix A)

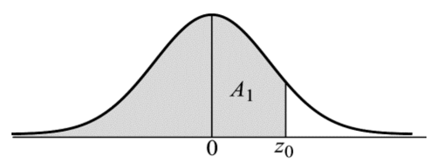
b. $P(\mu - \sigma < x < \mu + \sigma) = P(-1 < z < 1)$
$$= P(-1 < z < 0) + P(0 < z < 1)$$
$$= .3413 + .3413 = 2(.3413) = .6826$$
(from Table IV, Appendix A)

$P(\mu - 2\sigma < x < \mu + 2\sigma) = P(-2 < z < 2)$
$$= P(-2 < z < 0) + P(0 < z < 2)$$
$$= .4772 + .4772 = 2(.4772) = .9544$$
(from Table IV, Appendix A)

c. $P(x \le x_0) = .80$. Find $x_0$.

$$P(x \le x_0) = P\left(z \le \frac{x_0 - 300}{30}\right) = P(z \le z_0) = .80$$

$A_1 = .80 - .50 = .3000$



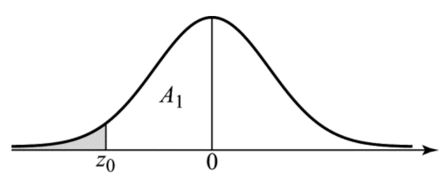

Looking up area .3000 in Table IV, $z_0 = .84$.

$$z_0 = \frac{x_0 - 300}{30} \Rightarrow .84 = \frac{x_0 - 300}{30} \Rightarrow x_0 = 325.2$$

$P(x \le x_0) = .10$. Find $x_0$.

$$P(x \le x_0) = P\left(z \le \frac{x_0 - 300}{30}\right) = P(z \le z_0) = .10$$

$A_1 = .50 - .10 = .4000$




Looking up area .4000 in Table IV, $z_0 = -1.28$.

$$z_0 = \frac{x_0 - 300}{30} \Rightarrow -1.28 = \frac{x_0 - 300}{30} \Rightarrow x_0 = 261.6$$

5.38 a. $P(x > 120) = P\left(z > \frac{120 - 105.3}{8}\right) = P(z > 1.84)$
$$= .5 - P(0 < z < 1.84) = .5 - .4671 = .0329$$
(Using Table IV, Appendix A)

b. $P(100 < x < 110) = P\left(\frac{100-105.3}{8} < z < \frac{110-105.3}{8}\right) = P(-.66 < z < .59)$

$= P(-.66 < z < 0) + P(0 < z < .59) = .2454 + .2224 = .4678$

(Using Table IV, Appendix A)

c. $P(x < a) = .25 \Rightarrow P\left(z < \frac{a-105.3}{8}\right) = P(z < z_o) = .25$

$A_1 = .5 - .25 = .25$

Looking up the area .25 in Table IV gives $z_o$ = -.67.

$z_o = -.67 = \frac{a-105.3}{8} \Rightarrow a = 8(-.67) + 105.3 = 99.94$

5.40 a. Using Table IV, Appendix A,

$P(x > 0) = P\left(z > \frac{0-5.26}{10}\right) = P(z > -.53) = .5000 + .2019 = .7019$

b. $P(5 < x < 15) = P\left(\frac{5-5.26}{10} < z < \frac{15-5.26}{10}\right) = P(-.03 < z < .97) = .0120 + .3340 = .3460$

c. $P(x < 1) = P\left(z < \frac{1-5.26}{10}\right) = P(z < -.43) = .5000 - .1664 = .3336$

d. $P(x < -25) = P\left(z < \frac{-25-5.26}{10}\right) = P(z < -3.03) = .5000 - .4988 = .0012$

Since the probability of seeing an average casino win percentage of -25% or smaller after 100 bets on black/red is so small (.0012), we would conclude that either the mean casino win percentage is not 5.26% but something smaller or the standard deviation of 10% is too small.

5.42 a. Let $x$ = carapace length of green sea turtle. Then $x$ has a normal distribution with $\mu = 55.7$ and $\sigma = 11.5$.

$P(x < 40) + P(x > 60) = P\left(z < \frac{40-55.7}{11.5}\right) + P\left(z > \frac{60-55.7}{11.5}\right)$

$= P(z < -1.37) + P(z > .37)$

$= .5 - .4147 + .5 - .1443 = .4410$

b. $P(x > L) = .10 \Rightarrow P\left(z > \frac{L-55.7}{11.5}\right) = P(z > z_o) = .10$

$A_1 = .5 - .10 = .40$

Looking up the area .40 in Table IV gives $z_o$ = 1.28.

$$z_o = 1.28 = \frac{L - 55.7}{11.5} \Rightarrow L = 1.28(11.5) + 55.7 = 70.42$$

5.44 a. If the player aims at the right goal post, he will score if the ball is less than 3 feet away from the goal post inside the goal (because the goalie is standing 12 feet from the goal post and can reach 9 feet). Using Table IV, Appendix A,

$$P(0 < x < 3) = P\left(0 < z < \frac{3-0}{3}\right) = P(0 < z < 1) = .3413$$

b. If the player aims at the center of the goal, he will be aimed at the goalie. In order to score, the player must place the ball more than 9 feet away from the goalie.
Using Table IV, Appendix A

$$P(x < -9) + P(x > 9) = P\left(z < \frac{-9-0}{3}\right) + P\left(z > \frac{9-0}{3}\right)$$
$$= P(z < -3) + P(z > 3) \approx .5 - .5 + .5 - .5 = 0$$

c. If the player aims halfway between the goal post and the goalie's reach, he will be aiming 1.5 feet from the goal post. Therefore, he will score if he hits from 1.5 feet to the left of where he is aiming to 1.5 feet to the right of where he is aiming. Using Table IV, Appendix A,

$$P(-1.5 < x < 1.5) = P\left(\frac{-1.5-0}{3} < z < \frac{1.5-0}{3}\right)$$
$$= P(-.5 < z < .5) = .1915 + .1915 = .3830$$

5.46 a. Let $x$ = rating of employee's performance. Then $x$ has a normal distribution with $\mu = 50$ and $\sigma = 15$. The top 10% get "exemplary" ratings.

$$P(x > x_o) = .10 \Rightarrow P\left(z > \frac{x_o - 50}{15}\right) = P(z > z_o) = .10$$

$$A_1 = .5 - .10 = .40$$

Looking up the area .40 in Table IV gives $z_o$ = 1.28.

$$z_o = 1.28 = \frac{x_o - 50}{15} \Rightarrow x_o = 1.28(15) + 50 = 69.2$$

b. Only 30% of the employees will get ratings lower than "competent".

$$P(x < x_o) = .30 \Rightarrow P\left(z < \frac{x_o - 50}{15}\right) = P(z < z_o) = .30$$

$$A_1 = .5 - .30 = .20$$

Looking up the area .20 in Table IV gives $z_o$ = -.52. The value of $z_o$ is negative because it is in the lower tail.

$$z_o = -.52 = \frac{x_o - 50}{15} \Rightarrow x_o = -.52(15) + 50 = 42.2$$

5.48 a. Using Table IV, Appendix A,

$$P(40 < x < 50) = P\left(\frac{40 - 37.9}{12.4} < z < \frac{50 - 37.9}{12.4}\right) = P(.17 < z < .98)$$
$$= .3365 - .0675 = .2690.$$

b. Using Table IV, Appendix A,

$$P(x < 30) = P\left(z < \frac{30 - 37.9}{12.4}\right) = P(z < -.64) = .5 - .2389 = .2611 .$$

c. We know that if $P(z_L < z < z_U) = .95$ , then $P(z_L < z < 0) + P(0 < z < z_U) = .95$ and

$P(z_L < z < 0) = P(0 < z < z_U) = .95 / 2 = .4750$ .

Using Table IV, Appendix A, $z_U$ = 1.96 and $z_L$ = -1.96.

$$P(x_L < x < x_U) = .95 \Rightarrow P\left(\frac{x_L - 37.9}{12.4} < z < \frac{x_U - 37.9}{12.4}\right) = .95$$

$$\Rightarrow \frac{x_L - 37.9}{12.4} = -1.96 \quad \text{and} \quad \frac{x_U - 37.9}{12.4} = 1.96$$

$$\Rightarrow x_L - 37.9 = -24.3 \text{ and } x_U - 37.9 = 24.3 \Rightarrow x_L = 13.6 \text{ and } x_U = 62.2$$

d. $P(z > z_0) = .10 \Rightarrow P(0 < z < z_0) = .4000$ . Using Table IV, Appendix A, $z_0$ = 1.28.

$$P(x > x_0) = .10 \Rightarrow \frac{x_0 - 37.9}{12.4} = 1.28 \Rightarrow x_0 - 37.9 = 15.9 \Rightarrow x_0 = 53.8 .$$

5.50 a. Let x = fill of container. Using Table IV, Appendix A,

$$P(x < 10) = P\left(z < \frac{10 - 10}{.2}\right) = P(z < 0) = .5$$

b. Profit = Price – cost – reprocessing fee = \$230 – \$20(10.6) – \$10
= \$230 – \$212 – \$10 = \$8.

c. If the probability of underfill is approximately 0, then Profit = Price – Cost.

$E$(Profit) = $E$(Price – Cost) = \$230 – $E$(Cost) = \$230 – \$20$E(x)$ = \$230 – \$20(10.5)

= \$230 – \$210 = \$20.

5.52 Let $x$ = load. From the problem, we know that the distribution of x is normal with a mean of 20.

$$P(10 < x < 30) = P\left(\frac{10-20}{\sigma} < z < \frac{30-20}{\sigma}\right) = P(-z_0 < z < z_0) = .95$$

First, we need to find $z_0$ such that $P(-z_0 < z < z_0) = .95$. Since we know that the center of the $z$ distribution is 0, half of the area or .95/2 = .475 will be between $-z_0$ and 0 and half will be between 0 and $z_0$.

We look up .475 in the body of Table IV, Appendix A to find $z_0$ = 1.96.
Thus,

$$\frac{30-20}{\sigma} = 1.96$$
$$\Rightarrow 1.96\sigma = 10 \Rightarrow \sigma = \frac{10}{1.96} = 5.102$$

5.54 Four methods for determining whether the sample data come from a normal population are:

1. Use either a histogram or a stem-and-leaf display for the data and note the shape of the graph. If the data are approximately normal, then the graph will be similar to the normal curve.
2. Compute the intervals $\bar{x} \pm s,\ \ \bar{x} \pm 2s,\ \ \bar{x} \pm 3s,$ and determine the percentage of measurements falling in each. If the data are approximately normal, the percentages will be approximately equal to 68%, 95%, and 100%, respectively.
3. Find the interquartile range, IQR, and the standard deviation, s, for the sample, then calculate the ratio IQR / $s$. If the data are approximately normal, then IQR / $s \approx 1.3$.
4. Construct a normal probability plot for the data. If the data are approximately normal, the points will fall (approximately) on a straight line.

5.56 In a normal probability plot, the observations in a data set are ordered from smallest to largest and then plotted against the expected $z$-scores of observations calculated under the assumption that the data come from a normal distribution. If the data are normally distributed, a linear or straight-line trend will result.

5.58 a. IQR = $Q_U - Q_L$ = 195 – 72 = 123

b. IQR/$s$ = 123/95 = 1.295

c. Yes. Since IQR is approximately 1.3, this implies that the data are approximately normal.

5.60 a. Using MINITAB, the stem-and-leaf display of the data is:

**Stem-and-Leaf Display: Data**

```
Stem-and-leaf of Data  N  = 28
Leaf Unit = 0.10

 2   1  16
 3   2  1
 7   3  1235
 11  4  0356
(4)  5  0399
 13  6  03457
 8   7  34
 6   8  2446
 2   9  47
```

The data are somewhat mound-shaped, so the data could be normally distributed.

b. Using MINITAB, the descriptive statistics are:

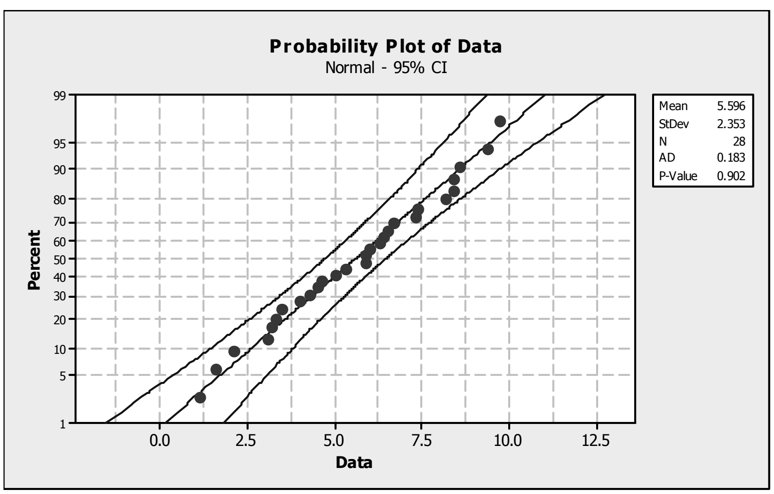
**Descriptive Statistics: Data**

```
Variable   N   Mean  StDev  Minimum     Q1  Median     Q3  Maximum
Data      28  5.596  2.353    1.100  3.625   5.900  7.375    9.700
```

From the printout, the standard deviation is $s = 2.353$.

c. From the printout, $Q_L = 3.625$ and $Q_U = 7.375$. The interquartile range is IQR = $Q_U - Q_L = 7.375 - 3.625 = 3.75$. If the data are approximately normal, then IQR / $s \approx 1.3$. For this data, IQR / $s = 3.75 / 2.353 = 1.397$. This is fairly close to 1.3, so the data could be normal.

d. Using MINITAB, the normal probability plot is:




Since the data are very close to a straight line, it indicates that the data could be normally distributed.

5.62 The histogram of the data is mound-shaped. It is somewhat skewed to the right, so it is not exactly symmetric. However, it is very close to a mound shaped distribution, so the engineers could use the normal probability distribution to model the behavior of shear strength for rock fractures.

5.64 a. We know that approximately 68% of the observations will fall within 1 standard deviation of the mean, approximately 95% will fall within 2 standard deviations of the mean, and approximately 100% of the observations will fall within 3 standard deviation of the mean. From the printout, the mean is 89.29 and the standard deviation is 3.18.

$\bar{x} \pm s \Rightarrow 89.29 \pm 3.18 \Rightarrow (86.11, 92.47)$. Of the 50 observations, 34 (or 34/50 = .68) fall between 86.11 and 92.47. This is close to what we would expect if the data were normally distributed.

$\bar{x} \pm 2s \Rightarrow 89.29 \pm 2(3.18) \Rightarrow 89.29 \pm 6.36 \Rightarrow (82.93, 95.65)$. Of the 50 observations, 48 (or 48/50 = .96) fall between 82.93 and 95.65. This is close to what we would expect if the data were normally distributed.

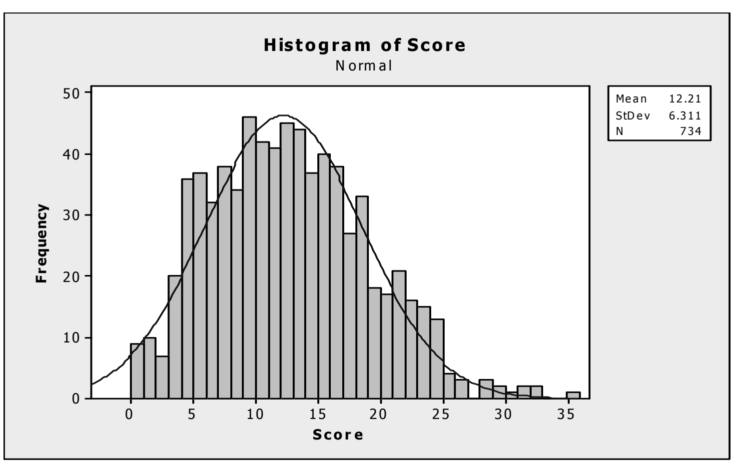
$\bar{x} \pm 3s \Rightarrow 89.29 \pm 3(3.18) \Rightarrow 89.29 \pm 9.54 \Rightarrow (79.75, 98.83)$. Of the 50 observations, 50 (or 50/50 = 1.00) fall between 79.75 and 98.83. This is close to what we would expect if the data were normally distributed.

The IQR = 4.84 and s = 3.18344. The ratio of the IQR and s is $\frac{IQR}{s} = \frac{4.84}{3.18344} = 1.52$. This is close to 1.3 that we would expect if the data were normally distributed.

Thus, there is evidence that the data are normally distributed.

b. If the data are normally distributed, the points will form a straight line when plotted using a normal probability plot. From the normal probability plot, the data points are close to a straight line. There is evidence that the data are normally distributed.

5.66 Using MINITAB, a histogram of the data with a normal curve drawn on the graph is:




From the graph, the data appear to be close to mound-shaped, so the data may be approximately normal.

5.68 **Distance**: To determine if the distribution of distances is approximately normal, we will run through the tests. Using MINITAB, the stem-and-leaf display is:

**Stem-and-Leaf Display: Distance**

```
Stem-and-leaf of Distance  N  = 40
Leaf Unit = 1.0


 1    28  3
 4    28  689
 10   29  011144
(11)  29  55556778889
 19   30  0000001112234
 6    30  59
 4    31  01
 2    31  68
```

From the stem-and-leaf display, the data look to be mound-shaped. The data may be normal.

Using MINITAB, the descriptive statistics are:

**Descriptive Statistics: Distance**

```
Variable   N    Mean  StDev  Minimum      Q1  Median      Q3  Maximum
Distance  40  298.95   7.53   283.20  294.60  299.05  302.00   318.90
```

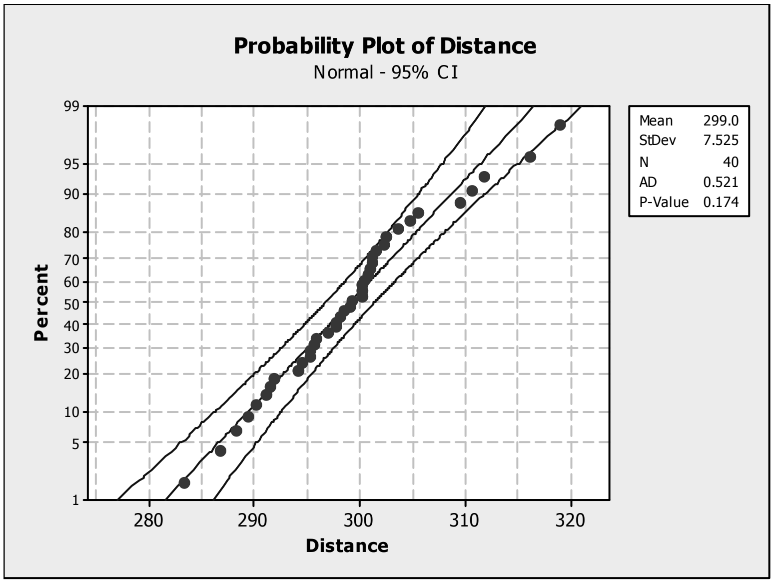
The interval $\bar{x} \pm s \Rightarrow 298.95 \pm 7.53 \Rightarrow (291.42,\ \ 306.48)$ contains 28 of the 40 observations. The proportion is 28 / 40 = .70. This is very close to the .68 from the Empirical Rule.

The interval $\bar{x} \pm 2s \Rightarrow 298.95 \pm 2(7.53) \Rightarrow 298.95 \pm 15.06 \Rightarrow (283.89,\ \ 314.01)$ contains 37 of the 40 observations. The proportion is 37 / 40 = .925. This is somewhat smaller than the .95 from the Empirical Rule.

The interval $\bar{x} \pm 3s \Rightarrow 298.95 \pm 3(7.53) \Rightarrow 298.95 \pm 22.59 \Rightarrow (276.36,\ \ 321.54)$ contains 40 of the 40 observations. The proportion is 40 / 40 = 1.00. This is very close to the .997 from the Empirical Rule. Thus, it appears that the data may be normal.

The lower quartile is $Q_L$ = 294.60 and the upper quartile is $Q_U$ = 302. The interquartile range is IQR = $Q_U - Q_L$ = 302 – 294.60 = 7.4. From the printout, s = 7.53. IQR / s = 7.4 / 7.53 = .983. This is somewhat less than the 1.3 that we would expect if the data were normal. Thus, there is evidence that the data may not be normal.

Using MINITAB, the normal probability plot is:




The data are very close to a straight line. Thus, it appears that the data may be normal.

From 3 of the 4 indicators, it appears that the distances come from an approximate normal distribution.

**Accuracy**: To determine if the distribution of accuracies is approximately normal, we will run through the tests. Using MINITAB, the stem-and-leaf display is:

**Stem-and-Leaf Display: Accuracy**

```
Stem-and-leaf of Accuracy  N  = 40
Leaf Unit = 1.0


 1   4  5
 1   4
 1   4
 2   5  0
 2   5
 3   5  4
 7   5  6777
 11  5  8999
 20  6  000001111
 20  6  2223333333
 10  6  4
 9   6  6667
 5   6  899
 2   7  0
 1   7  3
```

From the stem-and-leaf display, the data look to be skewed to the left. The data may not be normal.

Using MINITAB, the descriptive statistics are:

**Descriptive Statistics: Accuracy**

| Variable | N | Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|
| Accuracy | 40 | 61.970 | 5.226 | 45.400 | 59.400 | 61.950 | 64.075 | 73.000 |

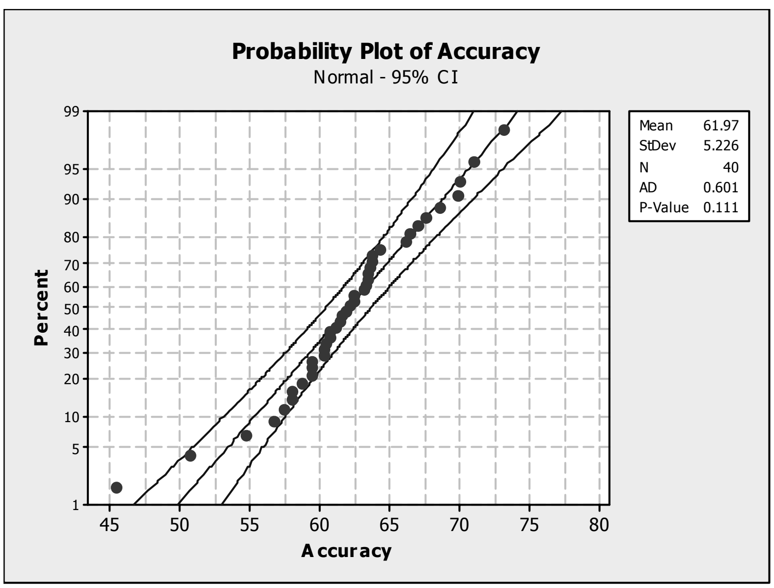
The interval $\bar{x} \pm s \Rightarrow 61.970 \pm 5.226 \Rightarrow (56.744,\ \ 67.196)$ contains 30 of the 40 observations. The proportion is 30 / 40 = .75. This is somewhat larger than the .68 from the Empirical Rule.

The interval $\bar{x} \pm 2s \Rightarrow 61.970 \pm 2(5.226) \Rightarrow 61.970 \pm 10.452 \Rightarrow (51.518,\ \ 72.422)$ contains 37 of the 40 observations. The proportion is 37 / 40 = .925. This is somewhat smaller than the .95 from the Empirical Rule.

The interval $\bar{x} \pm 3s \Rightarrow 61.970 \pm 3(5.226) \Rightarrow 61.970 \pm 15.678 \Rightarrow (46.292,\ \ 77.648)$ contains 39 of the 40 observations. The proportion is 39 / 40 = .975. This is somewhat smaller than the .997 from the Empirical Rule. Thus, it appears that the data may not be normal.

The lower quartile is $Q_L$ = 59.400 and the upper quartile is $Q_U$ = 64.075. The interquartile range is IQR = $Q_U - Q_L$ = 64.075 – 59.400 = 4.675. From the printout, $s$ = 5.226. IQR / $s$ = 4.675 / 5.226 = .895. This is less than the 1.3 that we would expect if the data were normal. Thus, there is evidence that the data may not be normal.

Using MINITAB, the normal probability plot is:




The data are not real close to a straight line. Thus, it appears that the data may not be normal.

From the 4 indicators, it appears that the accuracy values do not come from an approximate normal distribution.

**Index**: To determine if the distribution of driving performance index scores is approximately normal, we will run through the tests. Using MINITAB, the stem-and-leaf display is:

**Stem-and-Leaf Display: ZSUM**

```
Stem-and-leaf of ZSUM  N  = 40
Leaf Unit = 0.10

 1   1  1
 8   1  2233333
 18  1  4444445555
(4)  1  7777
 18  1  8899
 14  2  0
 13  2  22222
 8   2  5
 7   2  77
 5   2  8
 4   3  1
 3   3  2
 2   3  45
```

From the stem-and-leaf display, the data look to be skewed to the right. The data may not be normal.

Using MINITAB, the descriptive statistics are:

**Descriptive Statistics: ZSUM**

```
Variable   N   Mean  StDev  Minimum     Q1  Median     Q3  Maximum
ZSUM      40  1.927  0.660    1.170  1.400   1.755  2.218    3.580
```

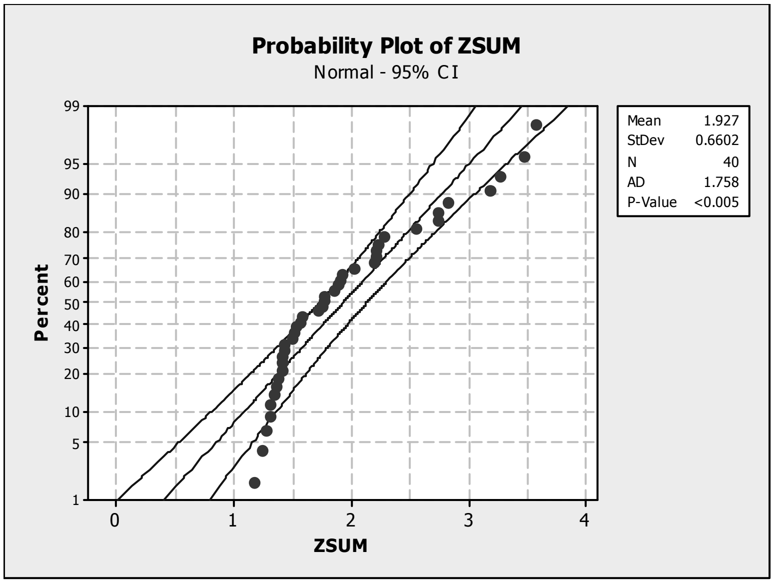
The interval $\bar{x} \pm s \Rightarrow 1.927 \pm .660 \Rightarrow (1.267, \quad 2.587)$ contains 30 of the 40 observations. The proportion is 30 / 40 = .75. This is somewhat larger than the .68 from the Empirical Rule.

The interval $\bar{x} \pm 2s \Rightarrow 1.927 \pm 2(.660) \Rightarrow 1.927 \pm 1.320 \Rightarrow (.607, \quad 3.247)$ contains 37 of the 40 observations. The proportion is 37 / 40 = .925. This is somewhat smaller than the .95 from the Empirical Rule.

The interval $\bar{x} \pm 3s \Rightarrow 1.927 \pm 3(.660) \Rightarrow 1.927 \pm 1.98 \Rightarrow (-.053, \quad 3.907)$ contains 40 of the 40 observations. The proportion is 40 / 40 = 1.000. This is slightly larger than the .997 from the Empirical Rule. Thus, it appears that the data may not be normal.

The lower quartile is $Q_L = 1.4$ and the upper quartile is $Q_U = 2.218$. The interquartile range is IQR = $Q_U - Q_L = 2.218 - 1.4 = .818$. From the printout, $s = .66$. IQR / s = .818 / .66 = 1.24. This is fairly close to the 1.3 that we would expect if the data were normal. Thus, there is evidence that the data may be normal.

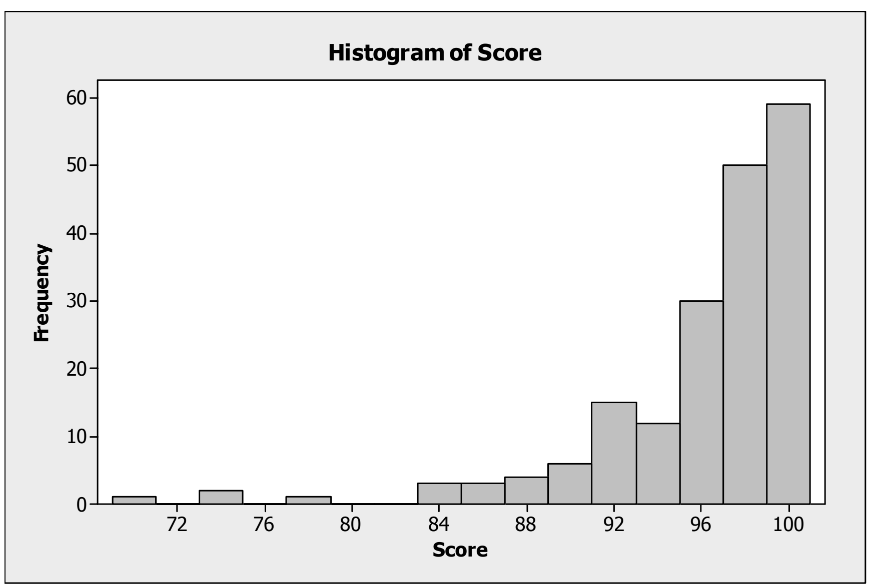
Using MINITAB, the normal probability plot is:




The data are not real close to a straight line. Thus, it appears that the data may not be normal.

From 3 of the 4 indicators, it appears that the driving performance index scores do not come from an approximate normal distribution.

5.70 To determine if the distribution of sanitation scores is approximately normal, we will run through the tests. Using MINITAB, the histogram of the data is:




From the histogram, the data look to be skewed to the left. The data may not be normal.

Using MINITAB, the descriptive statistics are:

**Descriptive Statistics: Score**

```
Variable    N    Mean  StDev  Minimum      Q1  Median      Q3  Maximum
Score     186  95.699  4.963   69.000  94.000  97.000  99.000  100.000
```

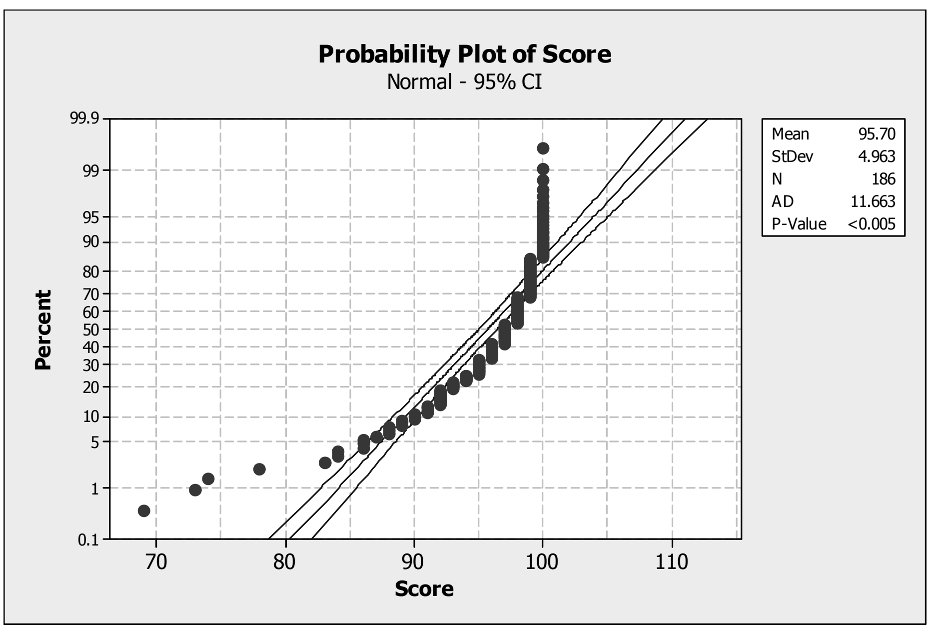
The interval $\bar{x} \pm s \Rightarrow 95.70 \pm 4.96 \Rightarrow (90.74,\ \ 100.66)$ contains 166 of the 186 observations. The proportion is 166 / 186 = .892. This is much larger than the .68 from the Empirical Rule.

The interval $\bar{x} \pm 2s \Rightarrow 95.70 \pm 2(4.96) \Rightarrow 95.70 \pm 9.92 \Rightarrow (85.78,\ \ 105.62)$ contains 179 of the 186 observations. The proportion is 179 / 186 = .962. This is somewhat larger than the .95 from the Empirical Rule.

The interval $\bar{x} \pm 3s \Rightarrow 95.70 \pm 3(4.96) \Rightarrow 95.70 \pm 14.88 \Rightarrow (80.82,\ \ 110.58)$ contains 182 of the 186 observations. The proportion is 182 / 186 = .978. This is somewhat smaller than the .997 from the Empirical Rule. Thus, it appears that the data may not be normal.

The lower quartile is $Q_L = 94$ and the upper quartile is $Q_U = 99$. The interquartile range is IQR = $Q_U - Q_L = 99 - 94 = 5$. From the printout, s = 4.963. IQR / $s$ = 5 / 4.963 = 1.007. This is not particularly close to the 1.3 that we would expect if the data were normal. Thus, there is evidence that the data may not be normal.

Using MINITAB, the normal probability plot is:




The data are not real close to a straight line. Thus, it appears that the data may not be normal.

From the 4 indicators, it appears that the sanitation scores do not come from an approximate normal distribution.

5.72 a. From the normal probability plot, it is very unlikely that the data are normally distributed. If the data are normal, the normal probability plot will form a straight line. For this normal probability plot, the data do not form a straight line. Thus, the data are not normal.

b. Since the data points corresponding to the largest *z*-scores are spread out further than the points corresponding to the smallest *z*-scores, the data are skewed to the right.

5.74 The binomial probability distribution is a discrete distribution. The random variable can take on only a limited number of values. The normal distribution is a continuous distribution. The random variable can take on an infinite number of values. To get a better estimate of probabilities for the binomial probability distribution using the normal distribution, we use the continuity correction factor.

5.76 a. $\mu = np = 100(.01) = 1.0$, $\sigma = \sqrt{npq} = \sqrt{100(.01)(.99)} = .995$

$\mu \pm 3\sigma \Rightarrow 1 \pm 3(.995) \Rightarrow 1 \pm 2.985 \Rightarrow (-1.985, 3.985)$

Since this interval does not fall in the interval (0, $n$ = 100), the normal approximation is not appropriate.

b. $\mu = np = 20(.6) = 12$, $\sigma = \sqrt{npq} = \sqrt{20(.6)(.4)} = 2.191$

$\mu \pm 3\sigma \Rightarrow 12 \pm 3(2.191) \Rightarrow 12 \pm 6.573 \Rightarrow (5.427, 18.573)$

Since this interval falls in the interval (0, $n$ = 20), the normal approximation is appropriate.

c. $\mu = np = 10(.4) = 4$, $\sigma = \sqrt{npq} = \sqrt{10(.4)(.6)} = 1.549$

$\mu \pm 3\sigma \Rightarrow 4 \pm 3(1.549) \Rightarrow 4 \pm 4.647 \Rightarrow (-.647, 8.647)$

Since this interval does not fall within the interval (0, $n$ = 10), the normal approximation is not appropriate.

d. $\mu = np = 1000(.05) = 50$, $\sigma = \sqrt{npq} = \sqrt{1000(.05)(.95)} = 6.892$

$\mu \pm 3\sigma \Rightarrow 50 \pm 3(6.892) \Rightarrow 50 \pm 20.676 \Rightarrow (29.324, 70.676)$

Since this interval falls within the interval (0, $n$ = 1000), the normal approximation is appropriate.

e. $\mu = np = 100(.8) = 80$, $\sigma = \sqrt{npq} = \sqrt{100(.8)(.2)} = 4$

$\mu \pm 3\sigma \Rightarrow 80 \pm 3(4) \Rightarrow 80 \pm 12 \Rightarrow (68, 92)$

Since this interval falls within the interval (0, $n$ = 100), the normal approximation is appropriate.

f. $\mu = np = 35(.7) = 24.5$, $\sigma = \sqrt{npq} = \sqrt{35(.7)(.3)} = 2.711$

$\mu \pm 3\sigma \Rightarrow 24.5 \pm 3(2.711) \Rightarrow 24.5 \pm 8.133 \Rightarrow (16.367, 32.633)$

Since this interval falls within the interval (0, $n$ = 35), the normal approximation is appropriate.

5.78 $\mu = np = 1000(.5) = 500$, $\sigma = \sqrt{npq} = \sqrt{1000(.5)(.5)} = 15.811$

a. Using the normal approximation,

$$P(x > 500) \approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > .03) = .5 - .0120 = .4880$$

(from Table IV, Appendix A)

b. $$P(490 \le x < 500) \approx P\left(\frac{(490-.5)-500}{15.811} \le z < \frac{(500-.5)-500}{15.811}\right)$$

$$= P(-.66 \le z < -.03) = .2454 - .0120 = .2334$$

(from Table IV, Appendix A)

c. $$P(x > 550) \approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > 3.19) \approx .5 - .5 = 0$$

(from Table IV, Appendix A)

5.80 a. For this exercise $n$ = 500 and $p$ = .5.

$\mu = np = 500(.5) = 250$ and $\sigma = \sqrt{npq} = \sqrt{500(.5)(.5)} = \sqrt{125} = 11.1803$

b. $$z = \frac{x-\mu}{\sigma} = \frac{240-250}{11.1803} = -.89$$

c. $$z = \frac{x-\mu}{\sigma} = \frac{270-250}{11.1803} = 1.79$$

d. $\mu \pm 3\sigma \Rightarrow 250 \pm 3(11.1803) \Rightarrow 250 \pm 33.5409 \Rightarrow (216.4591, 283.5409)$

Since the above is completely contained in the interval 0 to 500, the normal approximation is valid.

$$P(240 < x < 270) = P\left(\frac{(240+.5)-250}{11.1803} < z < \frac{(270-.5)-250}{11.1803}\right)$$

$$= P(-.85 < z < 1.74) = .3023 + .4591 = .7614$$

(Using Table IV, Appendix A)

5.82 a. Let $x$ = number of patients who experience serious post-laser vision problems in 100,000 trials. Then $x$ is a binomial random variable with n = 100,000 and $p$ =.01.

$$E(x) = \mu = np = 100{,}000(.01) = 1000.$$

b. $V(x) = \sigma^2 = npq = 100{,}000(.01)(.99) = 990$

c. $z = \dfrac{x-\mu}{\sigma} = \dfrac{950-1000}{\sqrt{990}} = \dfrac{-50}{31.4643} = -1.59$

d. $\mu \pm 3\sigma \Rightarrow 1000 \pm 3(31.4643) \Rightarrow 1000 \pm 94.3929 \Rightarrow (905.6071,\ 1{,}094.3929)$ Since the interval lies in the range 0 to 100,000, we can use the normal approximation to approximate the binomial probability.

$$P(x < 950) = P\left(z < \frac{(950-.5)-1000}{31.4643}\right) = P(z < -1.60) = .5 - .4452 = .0548$$

(Using Table IV, Appendix A)

5.84 a. $\mu = E(x) = np = 1000(.32) = 320$. This is the same value that was found in Exercise 4.66 a.

b. $\sigma = \sqrt{npq} = \sqrt{1000(.32)(1-.32)} = \sqrt{217.6} = 14.751$ This is the same value that was found in Exercise 4.66 b.

c. $z = \dfrac{x-\mu}{\sigma} = \dfrac{200.5-320}{14.751} = -8.10$

d. $\mu \pm 3\sigma \Rightarrow 320 \pm 3(14.751) \Rightarrow 320 \pm 44.253 \Rightarrow (275.474,\ 364.253)$

Since the above is completely contained in the interval 0 to 1000, the normal approximation is valid.

$$P(x \le 200) \approx P\left(z \le \frac{200.5-320}{14.751}\right) = P(z \le -8.10) \approx .5 - .5 = 0$$

(Using Table IV, Appendix A)

5.86 For this exercise, $n$ = 100 and $p$ = .4.

$\mu = np = 100(.4) = 40$ and $\sigma = \sqrt{npq} = \sqrt{100(.4)(.6)} = \sqrt{24} = 4.899$

$\mu \pm 3\sigma \Rightarrow 40 \pm 3(4.899) \Rightarrow 40 \pm 14.697 \Rightarrow (25.303,\ 54.697)$

Since the above is completely contained in the interval 0 to 100, the normal approximation is valid.

$$P(x < 50) = P\left(z < \frac{(50-.5)-40}{4.899}\right) = P(z < 1.94) = .5 + .4738 = .9738$$

(Using Table IV, Appendix A)

5.88 a. Let $x$ = number of abused women in a sample of 150. The random variable $x$ is a binomial random variable with $n$ = 150 and $p$ = 1/3. Thus, for the normal approximation,

$$\mu = np = 150(1/3) = 50 \text{ and } \sigma = \sqrt{npq} = \sqrt{150(1/3)(2/3)} = 5.7735$$
$$\mu \pm 3\sigma \Rightarrow 50 \pm 3(5.7735) \Rightarrow 50 \pm 17.3205 \Rightarrow (32.6795,\ 67.3205)$$

Since this interval lies in the range from 0 to $n$ = 150, the normal approximation is appropriate.

$$P(x > 75) \approx P\left(z > \frac{(75+.5)-50}{5.7735}\right) = P(z > 4.42) \approx .5 - .5 = 0$$

(Using Table IV, Appendix A.)

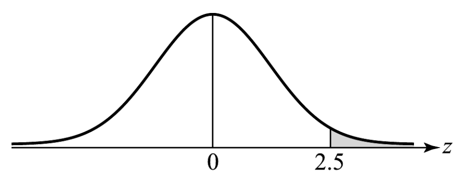
b. $$P(x < 50) \approx P\left(z < \frac{(50-.5)-50}{5.7735}\right) = P(z < -.09) \approx .5 - .0359 = .4641$$

c. $$P(x < 30) \approx P\left(z < \frac{(30-.5)-50}{5.7735}\right) = P(z < -3.55) \approx .5 - .5 = 0$$

Since the probability of seeing fewer than 30 abused women in a sample of 150 is so small ($p \approx 0$), it would be very unlikely to see this event.

5.90 a. Let $x$ equal the percentage of body fat in American men. The random variable $x$ is a normal random variable with $\mu$ = 15 and $\sigma$ = 2.

$$P(\text{Man is obese}) = P(x \ge 20) \approx P\left(z \ge \frac{20-15}{2}\right) = P(z \ge 2.5) = .5000 - .4938 = .0062$$



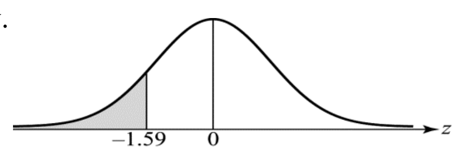

(Using Table IV in Appendix A.)

Let $y$ equal the number of men in the U.S. Army who are obese in a sample of 10,000. The random variable $y$ is a binomial random variable with $n$ = 10,000 and $p$ = .0062.

$$\mu \pm 3\sigma \Rightarrow np \pm 3\sqrt{npq} \Rightarrow 10{,}000(.0062) \pm 3\sqrt{10{,}000(.0062)(1-.0062)}$$
$$\Rightarrow 62 \pm 3(7.85) \Rightarrow (38.45,\ 85.55)$$

Since the interval does lie in the range 0 to 10,000, we can use the normal approximation to approximate the probability.

$$P(x < 50) \approx P\left(z < \frac{(50-.5)-62}{7.85}\right) \approx P(z < -1.59) = .5000 - .4441 = .0559$$




(Using Table IV in Appendix A.)

b. The probability of finding less than 50 obese Army men in a sample of 10,000 is .0559. Therefore, the probability of finding only 30 would even be smaller. Thus, it looks like the Army has successfully reduced the percentage of obese men since this did occur.

5.92 Let $x$ = number of patients who wait more than 30 minutes. Then $x$ is a binomial random variable with $n$ = 150 and $p$ = .5.

a. $\mu = np = 150(.5) = 75,\ \sigma = \sqrt{npq} = \sqrt{150(.5)(.5)} = 6.124$

$$P(x > 75) \approx P\left(z > \frac{(75+.5)-75}{6.124}\right) = P(z > .08) = .5 - .0319 = .4681$$

(from Table IV, Appendix A)

b. $$P(x > 85) \approx P\left(z > \frac{(85+.5)-75}{6.124}\right) = P(z > 1.71) = .5 - .4564 = .0436$$

(from Table IV, Appendix A)

c. $$P(60 < x < 90) \approx P\left(\frac{(60+.5)-75}{6.124} < z < \frac{(90-.5)-75}{6.124}\right)$$
$$= P(-2.37 < z < 2.37) = .4911 + .4911 = .9822$$

(from Table IV, Appendix A)

5.94 The exponential distribution is often called the **waiting time** distribution.

5.96 a. If $\theta$ = 1, $a$ = 1, then $e^{-a/\theta} = e^{-1} = .367879$

b. If $\theta$ = 1, $a$ = 2.5, then $e^{-a/\theta} = e^{-2.5} = .082085$

c. If $\theta$ = .4, $a$ = 3, then $e^{-a/\theta} = e^{-7.5} = .000553$

d. If $\theta$ = .2, $a$ = .3, then $e^{-a/\theta} = e^{-1.5} = .223130$

5.98 a. $P(x \le 4) = 1 - P(x > 4) = 1 - e^{-4/2.5} = 1 - e^{-1.6} = 1 - .201897 = .798103$

b. $P(x > 5) = e^{-5/2.5} = e^{-2} = .135335$

c. $P(x \le 2) = 1 - P(x > 2) = 1 - e^{-2/2.5} = 1 - e^{-.8} = 1 - .449329 = .550671$

d. $P(x > 3) = e^{-3/2.5} = e^{-1.2} = .301194$

5.100 With $\theta = 2$, $f(x) = \frac{1}{2}e^{-x/2}$ $(x > 0)$

$\mu = \sigma = \theta = 2$

a. $\mu \pm 3\sigma \Rightarrow 2 \pm 3(2) \Rightarrow 2 \pm 6 \Rightarrow (-4,\ 8)$
Since $\mu - 3\sigma$ lies below 0, find the probability that $x$ is more than $\mu + 3\sigma = 8$.

$P(x > 8) = e^{-8/2} = e^{-4} = .018316$ (using Table V in Appendix A)

b. $\mu \pm 2\sigma \Rightarrow 2 \pm 2(2) \Rightarrow 2 \pm 4 \Rightarrow (-2, 6)$

Since $\mu - 2\sigma$ lies below 0, find the probability that $x$ is between 0 and 6.

$$P(x < 6) = 1 - P(x \geq 6) = 1 - e^{-6/2} = 1 - e^{-3} = 1 - .049787 = .950213$$
(using Table V in Appendix A)

c. $\mu \pm .5\sigma \Rightarrow 2 \pm .5(2) \Rightarrow 2 \pm 1 \Rightarrow (1, 3)$

$$\begin{aligned} P(1 < x < 3) &= P(x > 1) - P(x > 3) \\ &= e^{-1/2} - e^{-3/2} = e^{-.5} - e^{-1.5} \\ &= .606531 - .223130 \\ &= .383401 \end{aligned}$$
(using Table V in Appendix A)

5.102 a. Let $x$ = time until the first critical part failure. Then $x$ has an exponential distribution with $\theta = .1$.

$$P(x \geq 1) = e^{-1/.1} = e^{-10} = .000045$$ (using Table V, Appendix A)

b. 30 minutes = .5 hours.
$$P(x < .5) = 1 - P(x \geq .5) = 1 - e^{-.5/.1} = 1 - e^{-5} = 1 - .006738 = .993262$$
(using Table V, Appendix A)

5.104 a. Let $x$ = time between component failures. Then $x$ has an exponential distribution with $\theta = 1000$.

$$P(1200 < x < 1500) = P(x > 1200) - P(x > 1500)$$
$$= e^{-1200/1000} - e^{-1500/1000} = e^{-1.2} - e^{-1.5} = .301194 - .223130 = .078064$$
(using Table V, Appendix A)

b. $P(x \geq 1200) = e^{-1200/1000} = e^{-1.2} = .301194$ (using Table V, Appendix A)

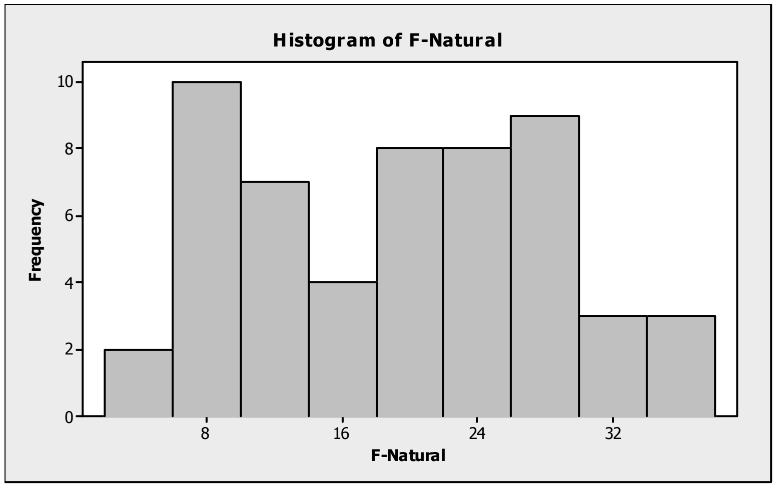
c. $$P(x < 1500 \mid x \geq 1200) = \frac{P(1200 \leq x < 1500}{P(x \geq 1200)} = \frac{.078064}{.301194} = .259182$$

5.106 a. Let $x$ = anthropogenic fragmentation index. Then $x$ has an exponential distribution with $\mu = \theta = 23$.

$$P(20 < x < 40) = P(x > 20) - P(x \geq 40) = e^{-20/23} - e^{-40/23} = .419134 - .1756730 = .243461$$

b. $$P(x < 50) = 1 - P(x \geq 50) = 1 - e^{-50/23} = 1 - .113732 = .886268$$

c. Using MINITAB, the histogram of the natural fragmentation index is:




An exponential distribution is skewed to the right. This histogram does not have that shape.

5.108 a. Let $x$ = life length of CD-ROM. Then $x$ has an exponential distribution with $\theta = 25{,}000$.
$R(t) = P(x > t) = e^{-t/25{,}000}$

b. $R(8{,}760) = P(x > 8{,}760) = e^{-8{,}760/25{,}000} = e^{-.3504} = .704406$

c. $S(t)$ = probability that at least one of two drives has a length exceeding $t$ hours

= 1 – probability that neither has a length exceeding $t$ hours

$= 1 - P(x_1 \le t)P(x_2 \le t) = 1 - [1 - P(x_1 > t)][1 - P(x_2 > t)]$

$= 1 - [1 - e^{-t/25{,}000}][1 - e^{-t/25{,}000}]$

$= 1 - [1 - 2e^{-t/25{,}000} + e^{-t/12{,}500}] = 2e^{-t/25{,}000} - e^{-t/12{,}500}$

d. $S(8{,}760) = 2e^{-8{,}760/25{,}000} - e^{-8{,}760/12{,}500} = 2(.704406) - .496188$
$= 1.408812 - .496188 = .912624$

e. The probability in part d is greater than that in part b. We would expect this. The probability that at least one of the systems lasts longer than 8,760 hours would be greater than the probability that only one system lasts longer than 8,760 hours.

5.110 Let $x$ = life length of a product. Then $x$ has an exponential distribution with $\mu = \theta$.

$$P(x > m) = e^{-m/\theta} = .5$$
$$\Rightarrow \frac{-m}{\theta} = \ln(.5) = -.6931$$
$$\Rightarrow m = .6931(\theta)$$

The median is equal to .6931 times the mean of the distribution.

5.112 a. A score on an IQ test probably follows a normal distribution.

b. Time waiting in line at a supermarket checkout counter probably follows an exponential distribution.

c. The amount of liquid dispensed into a can of soda probably follows a normal distribution.

d. The difference between SAT scores for tests taken at two different times probably follows a uniform distribution.

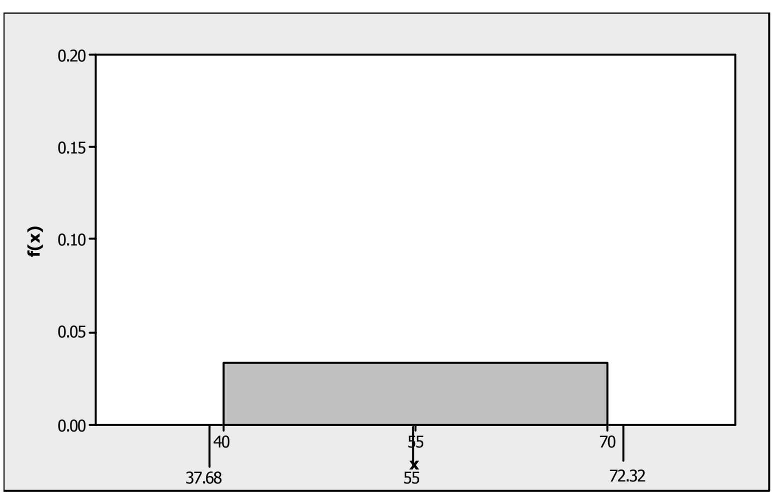
5.114 a. $f(x) = \frac{1}{d-c} = \frac{1}{70-40} = \frac{1}{30}$

$$f(x) = \begin{cases} \frac{1}{30} & (40 \le x \le 70) \\ 0 & \text{otherwise} \end{cases}$$

b. $\mu = \frac{c+d}{2} = \frac{40+70}{2} = 55$

$$\sigma = \frac{d-c}{\sqrt{12}} = \frac{70-40}{3.4641} = 8.660$$

c. $\mu \pm 2\sigma \Rightarrow 55 \pm 2(8.66) \Rightarrow (37.68, 72.32)$



d. $P(x \le 45) = (45 - 30)\frac{1}{30} = .167$

e. $P(x \ge 58) = (70 - 58)\frac{1}{30} = .4$

f. $P(x \le 100) = 1$, since this range includes all possible values of *x*.

g. $\mu \pm \sigma \Rightarrow 55 \pm 8.66 \Rightarrow (46.34,\ 63.66)$

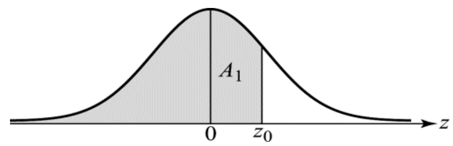
$P(\mu - \sigma < x < \mu + \sigma) = P(46.34 < x < 63.66) = (63.66 - 46.34)\frac{1}{30} = .577$

h. $P(x > 60) = (70 - 60)\frac{1}{30} = .333$

5.116 Using Table IV, Appendix A:

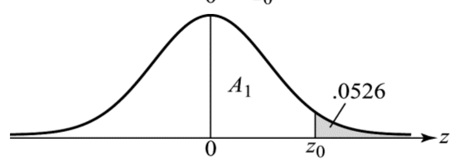
a. $P(z \le z_0) = .8708$

$A_1 = .8708 - .5 = .3708$
Looking up area .3708, $z_0$ = 1.13

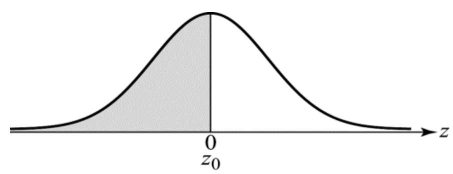



b. $P(z \ge z_0) = .0526$

$A_1 = .5 - .0526 = .4474$
Looking up area .4474, $z_0$ = 1.62

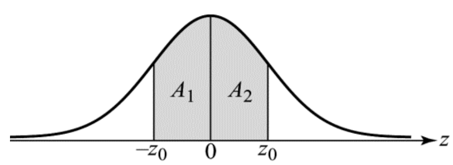



c. $P(z \le z_0) = .5 \Rightarrow z_0 = 0$



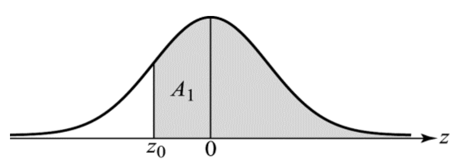

d. $P(-z_0 \le z \le z_0) = .8164$

$A_1 = A_2 = .8164 / 2 = .4082$
Looking up area .4082, $z_0$ = 1.33



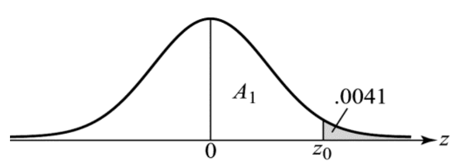

e. $P(z \ge z_0) = .8023$

$A_1 = .8023 - .5 = .3023$
Looking up area .3023, $z$ = .85
Since $z_0$ is to the left of 0, $z_0 = -.85$




f. $P(z \ge z_0) = .0041$

$A_1 = .5 - .0041 = .4959$
Looking up area .4959, $z_0$ = 2.64



5.118 Using Table IV, Appendix A:

a. $P(x \geq x_0) = .5$. Find $x_0$.

$$P(x \geq x_0) = P\left(z \geq \frac{x_0 - 40}{6}\right) = P(z \geq z_0) = .5 \Rightarrow z_0 = 0$$

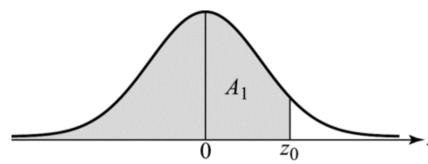
$$z_0 = \frac{x_0 - 40}{6} \Rightarrow 0 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40$$

b. $P(x \leq x_0) = .9911$. Find $x_0$.

$$P(x \leq x_0) = P\left(z \leq \frac{x_0 - 40}{6}\right) = P(z \leq z_0) = .9911$$

$A_1 = .9911 - .5 = .4911$
Looking up area .4911, $z_0 = 2.37$

$$z_0 = \frac{x_0 - 40}{6} \Rightarrow 2.37 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 54.22$$

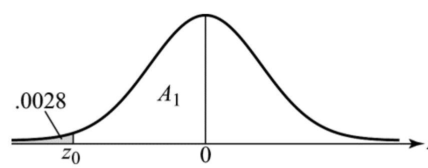



c. $P(x \leq x_0) = .0028$. Find $x_0$.

$$P(x \leq x_0) = P\left(z \leq \frac{x_0 - 40}{6}\right) = P(z \leq z_0) = .0028$$

$A_1 = .5 - .0028 = .4972$
Looking up area .4972, $z_0 = 2.77$
Since $z_0$ is to the left of 0, $z_0 = -2.77$

$$z_0 = \frac{x_0 - 40}{6} \Rightarrow -2.77 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 23.38$$

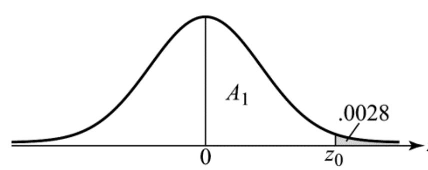



d. $P(x \geq x_0) = .0228$. Find $x_0$.

$$P(x \geq x_0) = P\left(z \geq \frac{x_0 - 40}{6}\right) = P(z \geq z_0) = .0228$$

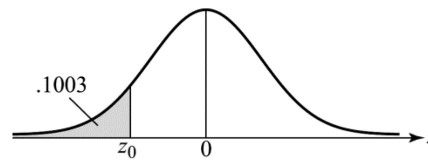
$A_1 = .5 - .0228 = .4772$
Looking up area .4772, $z_0 = 2.0$

$$z_0 = \frac{x_0 - 40}{6} \Rightarrow 2 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 52$$



e. $P(x \le x_0) = .1003$. Find $x_0$.

$$P(x \le x_0) = P\left(z \le \frac{x_0 - 60}{8}\right) = P(z \le z_0) = .1003$$

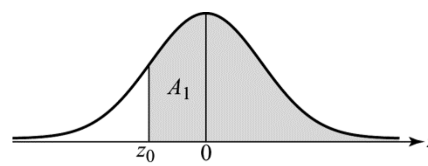



$A_1 = .5 - .1003 = .3997$
Looking up area .3997, $z = 1.28$.
Since $z_0$ is to the left of 0, $z_0 = -1.28$

$$z_0 = \frac{x_0 - 40}{6} \Rightarrow -1.28 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 32.32$$

f. $P(x \ge x_0) = .7995$. Find $x_0$.

$$P(x \ge x_0) = P\left(z \ge \frac{x_0 - 60}{8}\right) = P(z \ge z_0) = .7995$$




$A_1 = .7995 - .5 = .2995$
Looking up area .2995, $z = .84$.
Since $z_0$ is to the left of 0, $z_0 = -.84$

$$z_0 = \frac{x_0 - 40}{6} \Rightarrow -.84 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 34.96$$

5.120 a. $P(x \le 1) = 1 - P(x > 1) = 1 - e^{-1/3} = 1 - .716531 = .283469$ (using calculator)

b. $P(x > 1) = e^{-1/3} = .716531$

c. $P(x = 1) = 0$ ($x$ is a continuous random variable. There is no probability associated with a single point.)

d. $P(x \le 6) = 1 - P(x > 6) = 1 - e^{-6/3} = 1 - e^{-2} = 1 - .135335 = .864665$
(using Table V, Appendix A)

e. $P(2 \le x \le 10) = P(x \ge 2) - P(x > 10) = e^{-2/3} - e^{-10/3}$
$= .513417 - .035674 = .477743$ (using calculator)

5.122 a. For this problem, $c = 0$ and $d = 1$.

$$f(x) = \begin{cases} \dfrac{1}{d-c} = \dfrac{1}{1-0} & (0 \le x \le 1) \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{c+d}{2} = \frac{0+1}{2} = .5$$

$$\sigma^2 = \frac{(d-c)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12} = .0833$$

b. $P(.2 < x < .4) = (.4 - .2)(1) = .2$

c. $P(x > .995) = (1 - .995)(1) = .005$. Since the probability of observing a trajectory greater than .995 is so small, we would not expect to see a trajectory exceeding .995.

5.124 a. Let $x$ = change in SAT-MATH score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-19}{65}\right) = P(z \geq .48) = .5 - .1844 = .3156.$$

b. Let $x$ = change in SAT-VERBAL score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-7}{49}\right) = P(z \geq .88) = .5 - .3106 = .1894.$$

5.126 a. Let $x$ = weight of captured fish. Using Table IV, Appendix A,

$$P(1{,}000 < x < 1{,}400) = P\left(\frac{1{,}000-1{,}050}{375} < z < \frac{1{,}400-1{,}050}{375}\right) = P(-.13 < z < .93)$$
$$= .0517 + .3238 = .3755$$

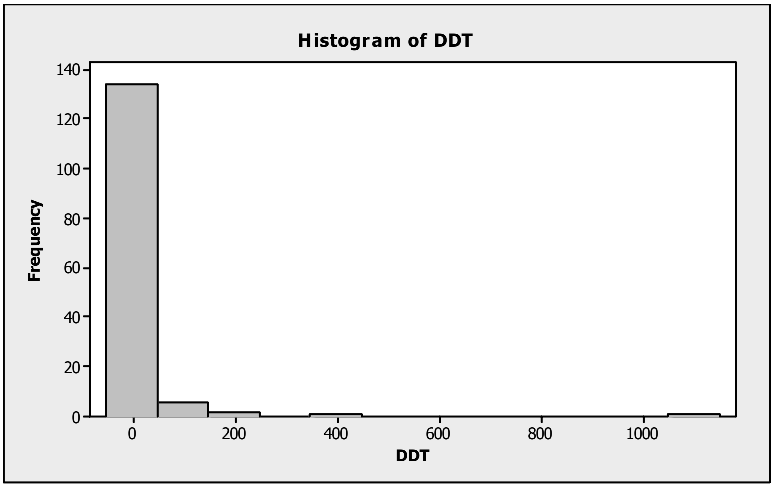
b. $$P(800 < x < 1{,}000) = P\left(\frac{800-1{,}050}{375} < z < \frac{1{,}000-1{,}050}{375}\right) = P(-.67 < z < -.13)$$
$$= .2486 - .0517 = .1969$$

c. $$P(x < 1{,}750) = P\left(z < \frac{1{,}750-1{,}050}{375}\right) = P(z < 1.87) = .5 + .4693 = .9693$$

d. $$P(x > 500) = P\left(z > \frac{500-1{,}050}{375}\right) = P(z > -1.47) = .5 + .4292 = .9292$$

e. $$P(x < x_o) = .95 \Rightarrow P\left(z < \frac{x_o - 1{,}050}{375}\right) = .95 \Rightarrow z = 1.645 = \frac{x_o - 1{,}050}{375}$$
$$\Rightarrow 616.875 = x_o - 1{,}050 \Rightarrow x_o = 1{,}666.875$$

f. We will look at the 4 methods for determining if the data are normal. First, we will look at a histogram of the data. Using MINITAB, the histogram of the fish DDT levels is:




From the histogram, the data appear to be skewed to the right. This indicates that the data may not be normal.

Next, we look at the intervals $\bar{x} \pm s$, $\bar{x} \pm 2s$, $\bar{x} \pm 3s$ . If the proportions of observations falling in each interval are approximately .68, .95, and 1.00, then the data are approximately normal. Using MINITAB, the summary statistics are:

**Descriptive Statistics: DDT**

| Variable | N | Mean | Median | StDev | Minimum | Maximum | Q1 | Q3 |
|---|---|---|---|---|---|---|---|---|
| DDT | 144 | 24.35 | 7.15 | 98.38 | 0.11 | 1100.00 | 3.33 | 13.00 |

$\bar{x} \pm s \Rightarrow 24.35 \pm 98.38 \Rightarrow (-74.03,\ \ 122.73)$ 138 of the 144 values fall in this interval. The proportion is .96. This is much greater than the .68 we would expect if the data were normal.

$\bar{x} \pm 2s \Rightarrow 24.35 \pm 2(98.38) \Rightarrow 24.35 \pm 196.76 \Rightarrow (-172.41,\ \ 221.11)$ 142 of the 144 values fall in this interval. The proportion is .986 This is much larger than the .95 we would expect if the data were normal.

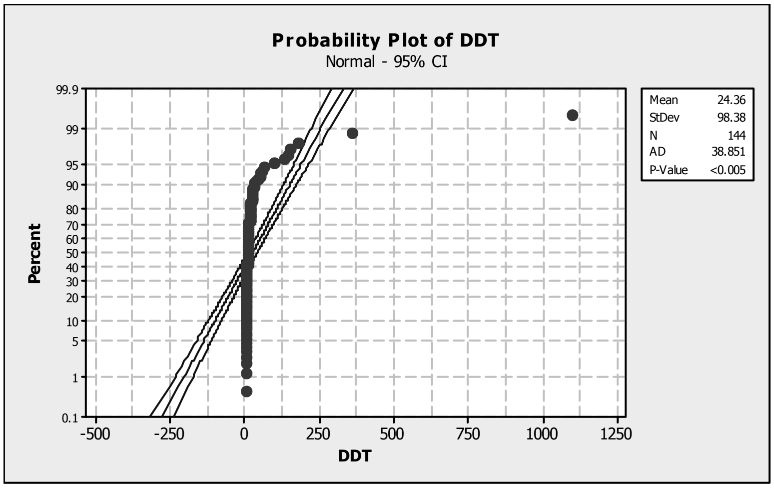
$\bar{x} \pm 3s \Rightarrow 24.35 \pm 3(98.38) \Rightarrow 24.35 \pm 295.14 \Rightarrow (-270.79,\ \ 319.49)$ 142 of the 144 values fall in this interval. The proportion is .986. This is somewhat lower than the 1.00 we would expect if the data were normal.

From this method, it appears that the data are not normal.

Next, we look at the ratio of the IQR to s. IQR = $Q_U - Q_L$ = 13.00 – 3.33 = 9.67.

$\frac{\text{IQR}}{\text{s}} = \frac{9.67}{98.38} = 0.098$ This is much smaller than the 1.3 we would expect if the data were normal. This method indicates the data are not normal.

Finally, using MINITAB, the normal probability plot is:




Since the data do not form a straight line, the data are not normal.

From the 4 different methods, all indications are that the fish DDT level data are not normal.

5.128 a. $\mu = np = 200(.5) = 100$

b. $\sigma = \sqrt{npq} = \sqrt{200(.5)(.5)} = \sqrt{50} = 7.071$

c. $z = \dfrac{x-\mu}{\sigma} = \dfrac{110-100}{7.071} = 1.41$

d. $P(x \le 110) \approx P\left(z \le \dfrac{(110+.5)-100}{7.071}\right) = P(z \le 1.48) = .5 + .4306 = .9306$

(Using Table IV, Appendix A)

5.130 Let $x$ = interarrival time between patients. Then $x$ is an exponential random variable with a mean of 4 minutes.

a. $P(x<1) = 1 - P(x \ge 1)$
$= 1 - e^{-1/4} = 1 - e^{-.25} = 1 - .778801 = .221199$ (Using Table V, Appendix A)

b. Assuming that the interarrival times are independent,

$P$(next 4 interarrival times are all less than 1 minute)
$= \{P(x<1)\}^4 = .221199^4 = .002394$

c. $P(x>10) = e^{-10/4} = e^{-2.5} = .082085$

5.132 a. For layer 2, $\mu = \frac{c+d}{2} = \frac{10+50}{2} = 30$ thousand dollars.

b. For layer 6, $\mu = \frac{c+d}{2} = \frac{500+1000}{2} = 750$ thousand dollars.

c. Let $x$ = marine loss for layer 2. $P(x > 30) = \frac{50-30}{50-10} = \frac{20}{40} = .5$

d. Let $x$ = marine loss for layer 6. $P(750 < x < 800) = \frac{800-750}{1000-500} = \frac{50}{500} = .1$

5.134 Let $x_1$ = score on the blue exam. Then $x_1$ is approximately normal with $\mu_1 = 53\%$ and $\sigma_1 = 15\%$.

$$P(x_1 < 20\%) = P\left(z < \frac{20-53}{15}\right) = P(z < -2.20) = .5 - .4861 = .0139$$

Let $x_2$ = score on the red exam. Then $x_2$ is approximately normal with $\mu_2 = 39\%$ and $\sigma_2 = 12\%$.

$$P(x_2 < 20\%) = P\left(z < \frac{20-39}{12}\right) = P(z < -1.58) = .5 - .4429 = .0571$$

Since the probability of scoring below 20% on the red exam is greater than the probability of scoring below 20% on the blue exam, it is more likely that a student will score below 20% on the red exam.

5.136 a. Let $x$ = gestation length. Using Table IV, Appendix A,

$$P(275.5 < x < 276.5) = P\left(\frac{275.5-280}{20} < z < \frac{276.5-280}{20}\right) = P(-.23 < z < -.18)$$
$$= .0910 - .0714 = .0196$$

b. Using Table IV, Appendix A,

$$P(258.5 < x < 259.5) = P\left(\frac{258.5-280}{20} < z < \frac{259.5-280}{20}\right) = P(-1.08 < z < -1.03)$$
$$= .3599 - .3485 = .0114$$

c. Using Table IV, Appendix A,

$$P(254.5 < x < 255.5) = P\left(\frac{254.5-280}{20} < z < \frac{255.5-280}{20}\right) = P(-1.28 < z < -1.23)$$
$$= .3997 - .3907 = .0090$$

d. If births are independent, then

$P$(baby 1 is 4 days early $\cap$ baby 2 is 21 days early $\cap$ baby 3 is 25 days early)

= $P$(baby 1 is 4 days early) $P$(baby 2 is 21 days early) $P$(baby 3 is 25 days early)

= .0196(.0114)(.0090) = .00000201.

5.138 Let $x$ = number of parents who condone spanking in 150 trials. Then $x$ is a binomial random variable with $n$ = 150 and $p$ = .6.

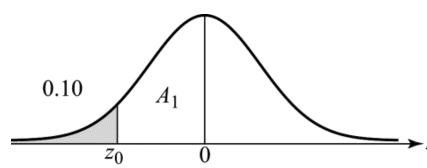
$$\mu = np = 150(.6) = 90$$

$$\sigma = \sqrt{npq} = \sqrt{150(.6)(.4)} = \sqrt{36} = 6$$

$$P(x \le 20) \approx P\left(z \le \frac{(20+.5)-90}{6}\right) = P(z \le -11.58) \approx 0$$

If, in fact, 60% of parents with young children condone spanking, the probability of seeing no more than 20 out of 150 parent clients who condone spanking is essentially 0. Thus, the claim made by the psychologist is either incorrect or the 60% figure is too high.

5.140 a. Using Table IV, Appendix A, with $\mu = 450$ and $\sigma = 40$,




$$P(x < x_0) = .10 \Rightarrow P\left(z < \frac{x_0 - 450}{40}\right)$$

$$= P(z < z_0) = .10$$

$A_1 = .5 - .10 = .4000$

Looking up the area .4000 in Table IV gives $z_0$ = 1.28. Since $z_0$ is to the left of 0, $z_0 = -1.28$.

$$z_0 = -1.28 = \frac{x_0 - 450}{40} \Rightarrow x_0 = 40(-1.28) + 450 = 398.8 \text{ seconds.}$$

5.142 a. Using Table IV, Appendix A, with $\mu = 24.1$ and $\sigma = 6.30$,

$$P(x \ge 20) = P\left(z \ge \frac{20 - 24.1}{6.30}\right) = P(z \ge -.65) = .2422 + .5 = .7422$$

b. $$P(x \le 10.5) = P\left(z \le \frac{10.5 - 24.1}{6.30}\right) = P(z \le -2.16) = .5 - .4846 = .0154$$

c. No. The probability of having a cardiac patient who participates regularly in sports or exercise with a maximum oxygen uptake of 10.5 or smaller is very small ($p$ = .0154). It is very unlikely that this patient participates regularly in sports or exercise.

5.144 a. Using Table IV, Appendix A, with $\mu = 99$ and $\sigma = 4.3$,

$$P(x < x_0) = .99 \Rightarrow P\left(z < \frac{x_0 - 99}{4.3}\right) = P(z < z_0) = .99$$

$A_1 = .99 - .5 = .4900$
Looking up the area .4900 in Table IV gives $z_0$ = 2.33. Since $z_0$ is to the right of 0, $z_0$ = 2.33.

$$z_0 = 2.33 = \frac{x_0 - 99}{4.3} \Rightarrow x_0 = 4.3(2.33) + 99 = 109.019$$

5.146 With $\mu = \theta = 30$, $f(x) = \frac{1}{30} e^{-x/30} \quad (x > 0)$

$P$(outbreaks within 6 years) $= P(x \le 6)$

$$= 1 - P(x > 6) = 1 - e^{-6/30} = 1 - e^{-.2} = 1 - .818731 = .181269$$

(using Table V, Appendix A)

5.148 a. Let $x_1$ = repair time for machine 1. Then $x_1$ has an exponential distribution with $\mu_1 = 1$ hour.

$P(x_1 > 1) = e^{-1/1} = e^{-1} = .367879$ (using Table V, Appendix A)

b. Let $x_2$ = repair time for machine 2. Then $x_2$ has an exponential distribution with $\mu_2 = 2$ hours.

$P(x_2 > 1) = e^{-1/2} = e^{-.5} = .606531$ (using Table V, Appendix A)

c. Let $x_3$ = repair time for machine 3. Then $x_3$ has an exponential distribution with $\mu_3 = .5$ hours.

$P(x_3 > 1) = e^{-1/.5} = e^{-2} = .135335$ (using Table V, Appendix A)

Since the mean repair time for machine 4 is the same as for machine 3, $P(x_4 > 1) = P(x_3 > 1) = .135335$.

d. The only way that the repair time for the entire system will not exceed 1 hour is if all four machines are repaired in less than 1 hour. Thus, the probability that the repair time for the entire system exceeds 1 hour is:

$P$(Repair time entire system exceeds 1 hour)

$$= 1 - P[(x_1 \le 1) \cap (x_2 \le 1) \cap (x_3 \le 1) \cap (x_4 \le 1)]$$
$$= 1 - P(x_1 \le 1)P(x_2 \le 1)P(x_3 \le 1)P(x_4 \le 1)$$
$$= 1 - (1 - .367879)(1 - .606531)(1 - .135335)(1 - .135335)$$
$$= 1 - (.632121)(.393469)(.864665)(.864665) = 1 - .185954 = .814046$$

5.150 a. Define $x$ = the number of serious accidents per month. Then $x$ has a Poisson distribution with $\lambda = 2$. If we define $y$ = the time between adjacent serious accidents, then $y$ has an exponential distribution with $\mu = 1/\lambda = 1/2$. If an accident occurs today, the probability that the next serious accident will **not** occur during the next month is:

$$P(y > 1) = e^{-1(2)} = e^{-2} = .135335$$

Alternatively, we could solve the problem in terms of the random variable $x$ by noting that the probability that the next serious accident will **not** occur during the next month is the same as the probability that the number of serious accident next month is zero, i.e.,

$$P(y > 1) = P(x = 0) = \frac{e^{-2}2^0}{0!} = e^{-2} = .135335$$

b. $P(x > 1) = 1 - P(x \le 1) = 1 - .406 = .594$

(Using Table III in Appendix A with $\lambda = 2$)

5.152 Let $x$ = weight of corn chip bag. Then $x$ is a normal random variable with $\mu = 10.5$ and $\sigma = .25$.

$$P(x > 10) = P\left(z > \frac{10 - 10.5}{.25}\right) = P(z > -2.00) = .4772 + .5000 = .9772$$

(Using Table IV, Appendix A)

Let $y$ = number of corn chip bags with more than 10 ounces. Then $x$ is a binomial random variable with $n$ = 1,500 and $p$ = .9772.

$$\mu = np = 1500(.9772) = 1465.8 \text{ and } \sigma = \sqrt{npq} = \sqrt{1500(.9772)(.0228)} = 5.781$$

97% of the 1500 chip bags is 1455

$$P(y \le 1455) = P\left(z \le \frac{(1455 + .5) - 1465.8}{5.781}\right) = P(z \le -1.78) = .5 - .4625 = .0375$$