Probability

Chapter **3**

3.2 The most basic outcomes of an experiment are sample points.

3.4 A Venn diagram is a pictorial method for presenting the sample space of an experiment.

3.6 An event is a specific collection of sample points.

3.8 Suppose that there are 5 people running for 2 openings on a school board. To find all the possible combinations of the two winners, we would take a combination of 5 people taken 2 at a time.

3.10 a. If the simple events are equally likely, then

$$P(1) = P(2) = P(3) = \cdots = P(10) = \frac{1}{10}$$

Therefore,

$$P(A) = P(4) + P(5) + P(6) = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10} = .3$$

$$P(B) = P(6) + P(7) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = .2$$

b. $$P(A) = P(4) + P(5) + P(6) = \frac{1}{20} + \frac{1}{20} + \frac{3}{20} = \frac{5}{20} = .25$$

$$P(B) = P(6) + P(7) = \frac{3}{10} + \frac{3}{10} = \frac{6}{10} = .3$$

3.12 a. $$\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 126$$

b. $$\binom{7}{2} = \frac{7!}{2!(7-2)!} = \frac{7!}{2!5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 21$$

c. $$\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4!0!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} = 1$$

d. $$\binom{5}{0} = \frac{5!}{0!(5-0)!} = \frac{5!}{0!5!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1$$

e. $$\binom{6}{5} = \frac{6!}{5!(6-5)!} = \frac{6!}{5!1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} = 6$$

3.14 a. The sample space for this experiment would consist of pairs of digits, indicating the result on each of the two dice.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

b. Each of the above sample points is equally likely, and each therefore has a probability of 1/36.

c. The probability of each event below can be obtained by counting the number of sample points which belong to the event and multiplying this amount by 1/36. This results in the following:

$$P(A) = \frac{1}{36} \qquad P(B) = \frac{18}{36} = \frac{1}{2} \qquad P(C) = \frac{6}{36} = \frac{1}{6}$$

$$P(D) = \frac{11}{36} \qquad P(E) = \frac{6}{36} = \frac{1}{6}$$

3.16 Each student will obtain slightly different proportions. However, the proportions should be close to $P(A) = 1/10$, $P(B) = 6/10$ and $P(C) = 3/10$.

3.18 It is stated that *P*(whale scream) = .03 and P(ship sound) = .14. A sound picked up by the acoustical equipment is more likely to be a sound of a passing ship than a whale scream because the probability of a ship sound is larger than the probability of a whale scream.

3.20 a. There are only two sample points for this experiment: white rhino and black rhino.

b. Use the relative frequencies as estimates of probabilities for the sample points. Thus,

P(white rhino) = 3,610/(3,610 + 11,330) = 3,610/14,940 = .242

P(black rhino) = 11,330/(3,610 + 11,330) = 11,330/14,940 = .758

3.22 a. *P*(beachgoer acquires gastroenteritis if he/she spends 10-minutes in the wet sand) = 6/1000 = .006.

b. *P*(beachgoer acquires gastroenteritis if he/she spends two hours in the wet sand) = 12/100 = .12.

c. *P*(beachgoer acquires gastroenteritis if he/she spends 10-minutes in the ocean water) = 7/1000 = .007.

d. *P*(beachgoer acquires gastroenteritis if he/she spends 70-minutes in the ocean water) = 7/100 = .07.

3.24 No. If the probability of rain this afternoon is .4, that means that of all the days with similar weather conditions as today, it rained on .4 or 40% of them.

3.26 *P*(museum uses big shows most often as performance measure) = 6/30 = 1/5 = .20.

3.28 a. *P*(student selected plagiarized on first essay) = 3/6 = ½ = .5.

b. Let P_i denote student i plagiarized and let N_i denote that student i did not plagiarized. The possible outcomes on the second essay are:

$N_1N_2N_3N_4N_5N_6$	$P_1N_2N_3N_4N_5N_6$	$N_1P_2N_3N_4N_5N_6$	$N_1N_2P_3N_4N_5N_6$
$N_1N_2N_3P_4N_5N_6$	$N_1N_2N_3N_4P_5N_6$	$N_1N_2N_3N_4N_5P_6$	$P_1P_2N_3N_4N_5N_6$
$P_1N_2P_3N_4N_5N_6$	$P_1N_2N_3P_4N_5N_6$	$P_1N_2N_3N_4P_5N_6$	$P_1N_2N_3N_4N_5P_6$
$N_1P_2P_3N_4N_5N_6$	$N_1P_2N_3P_4N_5N_6$	$N_1P_2N_3N_4P_5N_6$	$N_1P_2N_3N_4N_5P_6$
$N_1N_2P_3P_4N_5N_6$	$N_1N_2P_3N_4P_5N_6$	$N_1N_2P_3N_4N_5P_6$	$N_1N_2N_3P_4P_5N_6$
$N_1N_2N_3P_4N_5P_6$	$N_1N_2N_3N_4P_5P_6$	$P_1P_2P_3N_4N_5N_6$	$P_1P_2N_3P_4N_5N_6$
$P_1P_2N_3N_4P_5N_6$	$P_1P_2N_3N_4N_5P_6$	$P_1N_2P_3P_4N_5N_6$	$P_1N_2P_3N_4P_5N_6$
$P_1N_2P_3N_4N_5P_6$	$P_1N_2N_3P_4P_5N_6$	$P_1N_2N_3P_4N_5P_6$	$P_1N_2N_3N_4P_5P_6$
$N_1P_2P_3P_4N_5N_6$	$N_1P_2P_3N_4P_5N_6$	$N_1P_2P_3N_4N_5P_6$	$N_1P_2N_3P_4P_5N_6$
$N_1P_2N_3P_4N_5P_6$	$N_1P_2N_3N_4P_5P_6$	$N_1N_2P_3P_4P_5N_6$	$N_1N_2P_3P_4N_5P_6$
$N_1N_2P_3N_4P_5P_6$	$N_1N_2N_3P_4P_5P_6$	$P_1P_2P_3P_4N_5N_6$	$P_1P_2P_3N_4P_5N_6$
$P_1P_2P_3N_4N_5P_6$	$P_1P_2N_3P_4P_5N_6$	$P_1P_2N_3P_4N_5P_6$	$P_1P_2N_3N_4P_5P_6$
$P_1N_2P_3P_4P_5N_6$	$P_1N_2P_3P_4N_5P_6$	$P_1N_2P_3N_4P_5P_6$	$P_1N_2N_3P_4P_5P_6$
$N_1P_2P_3P_4P_5N_6$	$N_1P_2P_3P_4N_5P_6$	$N_1P_2P_3N_4P_5P_6$	$N_1P_2N_3P_4P_5P_6$
$N_1N_2P_3P_4P_5P_6$	$P_1P_2P_3P_4P_5N_6$	$P_1P_2P_3P_4N_5P_6$	$P_1P_2P_3N_4P_5P_6$
$P_1P_2N_3P_4P_5P_6$	$P_1N_2P_3P_4P_5P_6$	$N_1P_2P_3P_4P_5P_6$	$P_1P_2P_3P_4P_5P_6$

c. If it is just as likely that an ESL student plagiarizes on the second essay as not, then each of the above outcomes are equally likely. Since there are 64 possible outcomes, then each outcome has a probability of occurring of 1/64.

P(no more than one ESL student plagiarizes on second essay)

$$= P(N_1N_2N_3N_4N_5N_6) + P(P_1N_2N_3N_4N_5N_6) + P(N_1P_2N_3N_4N_5N_6) + P(N_1N_2P_3N_4N_5N_6) + P(N_1N_2N_3P_4N_5N_6) + P(N_1N_2N_3N_4P_5N_6) + P(N_1N_2N_3N_4N_5P_6)$$

= 1/64 + 1/64 + 1/64 + 1/64 + 1/64 + 1/64 + 1/64 = 7/64 = .109.

3.30 a. The sample points are:

Private / Bedrock / Below Limit
Private / Bedrock / Detect
Private / Unconsolidated / Below Limit
Private / Unconsolidated / Detect
Public / Bedrock / Below Limit
Public / Bedrock / Detect
Public / Unconsolidated / Below Limit
Public / Unconsolidated / Detect

b. The number of wells in each category and the associated probabilities are shown in the table:

Sample Points	**Frequency**	**Probability**
Private / Bedrock / Below Limit	81	81/223 = .363
Private / Bedrock / Detect	22	22/223 = .099
Private / Unconsolidated / Below Limit	0	0/223 = 0
Private / Unconsolidated / Detect	0	0/223 = 0
Public / Bedrock / Below Limit	57	57/223 = .256
Public / Bedrock / Detect	41	41/223 = .184
Public / Unconsolidated / Below Limit	15	15/223 = .067
Public / Unconsolidated / Detect	7	7/223 = .031
Total	**223**	**223/223 = 1**

c. The probability that a well has a detectible level of MTBE is

$$\frac{22+0+41+7}{223}=\frac{70}{223}=.314$$

About 1/3 of the time a well is chosen at random, it will have a detectible level of MTBE.

3.32 Suppose we label the 4 socks as B_1, B_2, N_1, and N_2. A list of all possible combinations of how the 4 socks could be paired is:

B_1, B_2 and N_1, N_2; B_1, N_1 and B_2, N_2; B_1, N_2 and B_2, N_1

Each of these three combinations are equally likely. Of these 3, only 1 has the socks matched correctly. Thus, the probability of matching the socks is only 1/3 and the probability of mismatching the socks is 2/3.

3.34 a. The simple events of this sample space could be represented by pairs where the first symbol would represent the gene obtained from the mother; the second symbol, the father.

$S = \{BB, Bb, bB, bb\}$

Each of these are equally likely, and the only way a child could have blue eyes would be if "*bb*" is the child's genetic pair, which has a probability of 1/4.

b. For convenience, let us say that it is the mother whose gene pair is *Bb*. The only possible simple events here would be:

$S = \{Bb, bb\}$

Again, each of these are equally likely, so that the probability of blue eyes in this case is 1/2.

c. Since the *BB* parent would donate a "*B*" gene, the child could not have blue eyes; the probability would be 0.

3.36 The intersection of 2 events *A* and *B* is the event that occurs if both *A* and *B* occur on a single performance of the experiment.

3.38 The Rule of Complements is: The sum of the probabilities of complementary events equals 1; that is, $P(A)+P(A^c)=1$.

3.40 Events A and B are mutually exclusive if $A\cap B$ contains no sample points; that is, if A and B have no sample points in common.

3.42 a. $P(B^c)=1-P(B)=1-.7=.3$

b. $P(A^c)=1-P(A)=1-.4=.6$

c. $P(A\cup B)=P(A)+P(B)-P(A\cap B)=.4+.7-.3=.8$

3.44 The experiment consists of rolling a pair of fair dice. The simple events are:

1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

Since each die is fair, each simple event is equally likely. The probability of each simple event is 1/36.

a. A: {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

B: {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

$A\cap B$: {(3, 4), (4, 3)}

$A\cup B$: {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (1, 6), (2, 5), (5, 2), (6, 1)}

A^c: {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 5), (3, 6), (4, 1), (4, 2), (4, 4), (4, 5), (4, 6), (5, 1), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

b. $P(A)=6\left(\frac{1}{36}\right)=\frac{6}{36}=\frac{1}{6}$

$P(B)=11\left(\frac{1}{36}\right)=\frac{11}{36}$

$P(A\cap B)=2\left(\frac{1}{36}\right)=\frac{2}{36}=\frac{1}{18}$

$P(A\cup B)=15\left(\frac{1}{36}\right)=\frac{15}{36}=\frac{5}{12}$

$P(A^c)=30\left(\frac{1}{36}\right)=\frac{30}{36}=\frac{5}{6}$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{11}{36} - \frac{1}{18} = \frac{6+11-2}{36} = \frac{15}{36} = \frac{5}{12}$

d. *A* and *B* are not mutually exclusive. To be mutually exclusive, $P(A \cap B)$ must be 0. Here, $P(A \cap B) = \frac{1}{18}$

3.46 a. $P(A^c) = P(E_3) + P(E_6) + P(E_8) = .2 + .3 + .03 = .53$

b. $P(B^c) = P(E_1) + P(E_7) + P(E_8) = .10 + .06 + .03 = .19$

c. $P(A^c \cap B) = P(E_3) + P(E_6) = .2 + .3 = .5$

d. $P(A \cup B) = P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7)$
$= .10 + .05 + .2 + .2 + .06 + .3 + .06 = .97$

e. $P(A \cap B) = P(E_2) + P(E_4) + P(E_5) = .05 + .2 + .06 = .31$

f. $P(A^c \cup B^c) = P(E_1) + P(E_7) + P(E_3) + P(E_6) + P(E_8) = .10 + .06 + .20 + .30 + .03 = .69$

g. No. *A* and *B* are mutually exclusive if $P(A \cap B) = 0$. Here, $P(A \cap B) = .31$.

3.48 Define the following events:

E_1: {3 heads}
E_2: {2 heads}
E_3: {1 heads}
E_4: {0 heads}

a. $A = E_1 \cup E_2 \cup E_3$

$P(A) = P(E_1) + P(E_2) + P(E_3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

b. $A = E_4^c$ $\qquad P(A) = 1 - P(E_4) = 1 - \frac{1}{8} = \frac{7}{8}$

3.50 a. $B \cap A$

b. A^c

c. $C \cup B$

d. $B^c \cap A^c$

3.52 Define the following event:

A: {Store violates the NIST scanner accuracy standard}

From the problem, we know $P(A) = 52/60 = .867$.

The probability that a randomly selected store does not violate the NIST scanner accuracy standard is $P(A^c) = 1 - P(A) = 1 - .867 = .133$.

3.54 a. There are 9 sample points for this experiment. They consist of all possible combinations for previous sleep stage and current stage. If we let the first event be the level of previous sleep stage and the second event be the level of current stage, the sample points are:

Non-REM, Non-REM
Non-REM, REM
Non-REM, Wake
REM, Non-REM
REM, REM
REM, Wake
Wake, Non-REM
Wake, REM
Wake, Wake

b. Reasonable probabilities would be the relative frequencies for each of the combinations. The total number of observations is $33,814 + 8,127 + 7,987 = 49,928$. Dividing each frequency by the total gives the following probabilities:

	Previous Sleep Stage			
Current Stage	Non-REM	REM	Wake	**Totals**
Non-REM	.6385	.0032	.0347	.6764
REM	.0069	.1524	.0035	.1628
Wake	.0318	.0072	.1218	.1607
Totals	.6772	.1628	.1600	

c. $P(\text{current REM}) = P(\text{Non-REM, REM}) + P(\text{REM, REM}) + P(\text{Wake, REM})$
$= .0069 + .1524 + .0035 = .1628$

d. $P(\text{previous Wake}) = P(\text{Wake, Non-REM}) + P(\text{Wake, REM}) + P(Wake, \text{Wake})$
$= .0347 + .0035 + .1218$

e. $P(\text{REM, Non-REM}) = .0032$

3.56 a. We can find reasonable probabilities the 4 sample points by dividing each frequency by the total sample size of 358. The estimates of the probabilities are:

Permit Drug at Home	*Less than 50*	*50 or more*	***Totals***
Yes	.475	.363	.838
No	.134	.028	.162
Totals	.609	.391	1.000

Define the following events:

A: {Provider permits home use of abortion drug}
B: {Provider has case load of less than 50 abortions}

$P(A) = .838$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .838 + .609 - .475 = .972$

c. $P(A \cap B) = .475$

3.58 a. Define the following events:
A: {Color code is 5}
B: {color code is 0}
C: {Model 2}

$$P(A) = \frac{35+50}{160} = \frac{85}{160} = .531$$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .531 + \frac{20+15}{160} - 0 = .531 + .219 = .750$

c. $P(C \cap B) = \frac{15}{160} = .094$

3.60 a. Define the following events:
A: {problem with absenteeism}
B: {problem with turnover}

From the exercise, we know that $P(A) = .55$, $P(B) = .41$, and $P(A \cap B) = .22$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = .55 + .41 - .22 = .74$$

b. $P(A^c) = 1 - P(A) = 1 - .55 = .45$

c. $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - .74 = .26$

3.62 There are a total of 6 x 6 x 6 = 216 possible outcomes to the throwing of 3 fair dice. A partition is a set of 3 numbers that sum to a particular number.

There are 6 partitions for a sum of 9 on 3 dice:

(1, 2, 6), (1, 3, 5), (1, 4, 4), (2, 2, 5), (2, 3, 4), (3, 3, 3).

However, some of these partitions can be obtained in more than one way if we let the order of the numbers (corresponding to particular die) matter. The total set of possible outcomes that sum to 9, where order matters is:

(1, 2, 6),	(1, 3, 5),	(1, 4, 4),	(2, 2, 5),	(2, 3, 4),	(3, 3, 3),
(1, 6, 2),	(1, 5, 3),	(4, 1, 4),	(2, 5, 2),	(2, 4, 3),	
(2, 1, 6),	(3, 1, 5),	(4, 4, 1),	(5, 2, 2),	(3, 2, 4),	
(2, 6, 1),	(3, 5, 1),			(3, 4, 2),	
(6, 1, 2),	(5, 1, 3),			(4, 2, 3),	
(6, 2, 1),	(5, 3, 1),			(4, 3, 2)	

There are a total of 25 sample points. Thus, the probability of getting a sum of 9 is 25 / 216 = .116.

There are also 6 partitions for a sum of 10 on 3 dice:

(1, 3, 6), (1, 4, 5), (2, 2, 6), (2, 3, 5), (2, 4, 4), (3, 3, 4).

Again, some of these partitions can be obtained in more than one way if we let the order of the numbers (corresponding to particular die) matter. The total set of possible outcomes that sum to 10, where order matters is:

(1, 3, 6),	(1, 4, 5),	(2, 2, 6),	(2, 3, 5),	(2, 4, 4),	(3, 3, 4),
(1, 6, 3),	(1, 5, 4),	(2, 6, 2),	(2, 5, 3),	(4, 2, 4),	(3, 4, 3),
(3, 1, 6),	(4, 1, 5),	(6, 2, 2),	(3, 2, 5),	(4, 4, 2),	(4, 3, 3),
(3, 6, 1),	(4, 5, 1),		(3, 5, 2),		
(6, 1, 3),	(5, 1, 4),		(5, 2, 3),		
(6, 3, 1),	(5, 4, 1),		(5, 3, 2)		

There are a total of 27 sample points. Thus, the probability of getting a sum of 10 is 27 / 216 = .125.

3.64 a. The Multiplicative Rule of Probability for two independent events is:

$$P(A \cap B) = P(A)P(B)$$

b. The Multiplicative Rule of Probability for any two events is:

$$P(A \cap B) = P(B \mid A)P(A) = P(A \mid B)P(B)$$

3.66 a. "Dependent events are always mutually exclusive" is not true. Suppose we define an experiment as follows. Two fair coins are tossed and the results are recorded. The possible outcomes are (*H*,*H*), (*H*, *T*), (*T*, *H*), (*T*, *T*). If the coins are fair, then these outcomes are equally likely with probabilities ¼. Define the following events:

A: {Both outcomes are *H*}
B: {Both outcomes are the same}

Then $P(A)=1/4$ and $P(B)=1/2$. The event $A\cap B$ is the event that both outcomes are *H* or event *A*. Thus, $P(A\cap B)=1/4$. Since this probability is not 0, events *A* and *B* are not mutually exclusive. However, events *A* and *B* are dependent:

$P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4}{1/2}=\dfrac{1}{2}$ and this does not equal $P(A)=1/4$, so events *A* and *B* are dependent.

Thus, dependent events are **not** always mutually exclusive.

b. "Mutually exclusive events are always dependent" is true. Suppose we define a third event:

C: {Both outcomes are T}

Now, events *A* and *C* are mutually exclusive because $P(A\cap C)=0$. Also,

$P(C\mid A)=\dfrac{P(C\cap A)}{P(A)}=\dfrac{0}{1/4}=0$ and this does not equal $P(C)=1/4$. Thus, events *A* and *C* are dependent.

For any two mutually exclusive events *A* and *C*, $P(A\cap C)=0$. Thus, $P(C\mid A)=P(A\mid C)=0$. As long as $P(A) > 0$ and $P(C) > 0$, then $P(C\mid A)\neq P(C)$ and $P(A\mid C)\neq P(A)$.

c. "Independent events are always mutually exclusive" is not true. Suppose we define 2 additional events:

D: {Both outcomes are different}
E: {The first coin is a *T*}

Now, $P(D)=1/4=1/2$, $P(E)=2/4=1/2$, and $P(D\cap E)=1/4$. Thus, since $P(D\cap E)=1/4$ which is not 0, *D* and *E* are not mutually exclusive. However, *D* and *E* are independent:

$P(D\mid E)=\dfrac{P(D\cap E)}{P(E)}=\dfrac{1/4}{1/2}=\dfrac{1}{2}$ and this does equal $P(D)=1/2$. Thus, events *D* and *E* are independent. Therefore, events *D* and *E* are independent but not mutually exclusive.

3.68 a. $P(A\cap B)=P(A\mid B)P(B)=.6(.2)=.12$

b. $P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{.12}{.4}=.3$

3.70 Since A, B, and C are all mutually exclusive, we know that

$$P(A \cap B) = P(A \cap C) = P(B \cap C) = 0$$

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .55 - 0 = .85$

b. $P(A \cap B) = 0$

c. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{.55} = 0$

d. $P(B \cup C) = P(B) + P(C) - P(B \cap C) = .55 + .15 - 0 = .70$

e. No, B and C are not independent events. If B and C are independent events, then

$P(B \mid C) = P(B)$. From the problem, we know $P(B) = .55$.

$P(B \mid C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{0}{.15} = 0$. Thus, since $P(B \mid C) \neq P(B)$, events B and C are not independent.

3.72 a. $P(A) = P(E_1) + P(E_3) = .22 + .15 = .37$

b. $P(B) = P(E_2) + P(E_3) + P(E_4) = .31 + .15 + .22 = .68$

c. $P(A \cap B) = P(E_3) = .15$

d. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.15}{.68} = .221$

e. $P(B \cap C) = 0$

f. $P(C \mid B) = \dfrac{P(B \cap C)}{P(B)} = \dfrac{0}{.68} = 0$

g. $P(A)P(B) = .37(.68) = .2516 \neq P(A \cap B) = .15$. Thus, A and B are not independent.

$P(A)P(C) = .37(.32) = .1184 \neq P(A \cap C) = .22$. Thus, A and C are not independent.

$P(B)P(C) = .68(.32) = .2176 \neq P(B \cap C) = 0$. Thus, B and C are not independent.

3.74 a. $P(A \cap B) = 0 \Rightarrow A$ and C are mutually exclusive.
$P(B \cap C) = 0 \Rightarrow B$ and C are mutually exclusive.

b. $P(A) = P(1) + P(2) + P(3) = .20 + .05 + .30 = .55$
$P(B) = P(3) + P(4) = .30 + .10 = .40$

$P(C) = P(5) + P(6) = .10 + .25 = .35$

$P(A \cap B) = P(3) = .30$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.30}{.40} = .75$$

A and *B* are independent if $P(A \mid B) = P(A)$. Since $P(A \mid B) = .75$ and $P(A) = .55$, *A* and *B* are not independent.

Since *A* and *C* are mutually exclusive, they are not independent. Similarly, since *B* and *C* are mutually exclusive, they are not independent.

c. Using the probabilities of simple events,
$P(A \cup B) = P(1) + P(2) + P(3) + P(4) = .20 + .05 + .30 + .10 = .65$

Using the additive rule,
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = .55 + .40 - .30 = .65$

Using the probabilities of simple events,
$P(A \cup C) = P(1) + P(2) + P(3) + P(5) + P(6) = .20 + .05 + .30 + .10 + .25 = .90$

Using the additive rule,
$P(A \cup C) = P(A) + P(C) - P(A \cap C) = .55 + .35 - 0 = .90$

3.76 a. $P(A) = .68$ indicates that event A is the event that an American adult has a public library card.

$P(A \mid B) = .62$ indicates that event B is the event that an American adult is male.

$P(A \mid C) = .73$ indicates that event C is the event that an American adult is female.

b. We are given that $P(B) = P(C) = .50$. Thus,
$P(C \cap A) = P(A \mid C)P(C) = .73(.50) = .365$.

3.78 a. Define the following events:
A: {Student choose stated option}
B: {Student did not choose stated option}
C: {Emotion state is Guilt}
D: {Emotion state is Anger}
E: {Emotion state is Neutral}

$$P(A \mid C) = \frac{45}{57} = .789$$

b. $$P(D \mid B) = \frac{50}{111} = .450$$

c. $P(A) = \frac{60}{171} = .351$ and $P(A \mid C) = \frac{45}{57} = .789$. Since these 2 probabilities are not the same, the events {repair the car} and {guilty state} are not independent.

3.80 Define the following events:
A: {internet user has wireless connection}
B: {internet user uses Twitter}

From the exercise, $P(A) = .54$ and $P(B \mid A) = .25$.

$P(A \cap B) = P(B \mid A)P(A) = .54(.25) = .135$

3.82 Define the following events:

A: {Pottery is painted}

B: {Pottery is painted in curvilinear decoration}

C: {Pottery is painted in geometric decoration }

D: {Pottery is painted in naturalistic decoration }

a. $P(A) = 183/837 = .219$

b. $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{14/837}{183/837} = \frac{14}{183} = .077$

3.84 a. Define the following events:
A: {previous stage is Wake}
B: {current stage is REM}
C: {previous stage is REM}
D: {current stage is Wake}

$P(B \mid A) = \frac{175}{7987} = .022$

b. $P(A^c \mid B) = \frac{346 + 7609}{346 + 7609 + 175} = \frac{7955}{8130} = .978$

c. $P(C \cap B) = \frac{7609}{33{,}814 + 8127 + 7987} = \frac{7609}{49{,}928} = .152$. Since this probability is not 0, the events {previous stage is REM} and {current stage is REM} are not mutually exclusive.

d. $P(B \mid C) = \frac{7609}{8127} = .936$. Also, $P(B) = \frac{346 + 7609 + 175}{49{,}928} = \frac{8130}{49{,}928} = .163$. Since $P(B \mid C) \neq P(B)$, the events {previous stage is REM} and {current stage is REM} are not independent.

e. $P(D \mid A) = \frac{6079}{7987} = .761$. Also, $P(D) = \frac{1588 + 358 + 6079}{49{,}928} = \frac{8025}{49{,}928} = .161$. Since $P(D \mid A) \neq P(D)$, the events {previous stage is Wake} and {current stage is Wake} are not independent.

3.86 Define the following events:

F: {Fight}
N: {No fight}
W: {Initiator Wins}
T: {No Clear Winner}
L: {Initiator Loses}

a. $P(W \mid F) = \frac{P(W \cap F)}{P(F)} = \frac{26/167}{64/167} = \frac{26}{64} = .406$

b. $P(W \mid N) = \frac{P(W \cap N)}{P(N)} = \frac{80/167}{103/167} = \frac{80}{103} = .777$

c. Two events are independent if $P(W \mid N) = P(W)$.

$P(W) = \frac{106}{167} = .635$ which does not equal $P(W \mid N) = .777$. Thus, "no fight" and "initiator wins" are not independent.

3.88 a. Define the following events:
A: {successful regime change}
B: {mission extended to support weak Iraq government}

$P(A^c) = 1 - P(A) = 1 - .70 = .30$

b. $P(A \mid B) = .26$

c. $P(A \cap B) = P(A \mid B)P(B) = .26(.55) = .143$

3.90 Define the following events:

I: {Intruder}
N: {No intruder}
A: {System A sounds alarm}
B: {System B sounds alarm}

a. From the problem:

$P(A \mid I) = .9$

$P(B \mid I) = .95$

$P(A \mid N) = .2$

$P(B \mid N) = .1$

b. We know that events A and B are independent. Thus,

$P(A \cap B \mid I) = P(A \mid I)P(B \mid I) = .9(.95) = .855$

c. $P(A \cap B \mid N) = P(A \mid N)P(B \mid N) = .2(.1) = .02$

d. $P(A \cup B \mid I) = P(A \mid I) + P(B \mid I) - P(A \cap B \mid I) = .9 + .95 - .855 = .995$

3.92 a. See the table below. I have numbered the 12 edges.

A 7 1	**A** 10 3	**N** 5
N 8 2	**F** 11 4	**F** 6
N 9	**F** 12	**F**

Edge 1 is A-N
Edge 2 is N-N
Edge 3 is F-A
Edge 4 is F-F
Edge 5 is F-N
Edge 6 is F-F
Edge 7 is A-A
Edge 8 is F-N
Edge 9 is F-N
Edge 10 is A-N
Edge 11 is F-F
Edge 12 is F-F

b. From the list in part a, there are only 8 F-edges. They are:

Edge 3 is F-A
Edge 4 is F-F
Edge 5 is F-N
Edge 6 is F-F
Edge 8 is F-N
Edge 9 is F-N
Edge 11 is F-F
Edge 12 is F-F

A summary of these edges is:

Edge	Frequency
F-A	1
F-N	3
F-F	4
Total	8

c. If an F-edge is selected at random, then the probability of selecting an F-A edge is equal to its relative frequency. Thus, $P(F - A)$ = 1/8 = .125

d. If an F-edge is selected at random, then the probability of selecting an F-N edge is equal to its relative frequency. Thus, $P(F - N) = 3/8 = .375$

3.94 a. Let N = normal cell and M = mutant cell. The possible pedigrees would be: NN, NM, MN, and MM.

b. If each "daughter cell" is equally likely to be normal or mutant, then $P(N) = P(M) = .5$. In addition, the probability of each of the possible pedigrees listed in part a is .5(.5) = .25. The probability that a normal cell that divides into two offspring will result in at least one mutant cell is:
$P(\text{at least one mutant cell}) = P(NM) + P(MN) + P(MM) = .25 + .25 + .25 = .75.$

c. Now, $P(M) = .2$ and $P(N) = 1 - .2 = .8.$

$P(NN) = .8(.8) = .64,\ P(NM) = .8(.2) = .16,\ P(MN) = .2(.8) = .16,$ and $P(MM) = .2(.2) = .04.$

$P(\text{at least one mutant cell}) = P(NM) + P(MN) + P(MM) = .16 + .16 + .04 = .36.$

d. If the first generation resulted in NN, then each of these first generation normal cells have 4 possible pedigrees, NN, NM, MN, and MM. Since there are 4 possible pedigrees for the 1^{st} first generation normal cell and 4 possible pedigrees for the 2^{nd} first generation normal cell, there are a total of 4(4)=16 possible second generation pedigrees from a first generation NN. They are:

NNNN NNNM NNMN NMNN MNNN NNMM NMNM NMMN MNNM

MNMN MMNN NMMM MNMM MMNM MMMN MMMM

If the first generation resulted in NM, then the first generation normal cell has 4 possible pedigrees, NN, NM, MN, and MM, and the first generation mutant cell has one possible pedigree, MM. Thus, there are a total of 4(1) = 4 possible second generation pedigrees from a first generation NM. They are:

NNMM NMMM MNMM MMMM

If the first generation resulted in MN, then the first generation mutant cell has one possible pedigree, MM, and the first generation normal cell has 4 possible pedigrees, NN, NM, MN, and MM. Thus, there are a total of 4(1) = 4 possible second generation pedigrees from a first generation NM. They are:

MMNN MMNM MMMN MMMM

If the first generation resulted in MM, then each first generation mutant cell has one possible pedigree, MM. Thus, there is only one possible second generation pedigree from a first generation MM. It is:

MMMM

e. If each "daughter" cell is equally likely to be mutant or normal, we can find the probabilities of all second generation pedigrees. The probability that the first generation pedigree is NN is 1/4. There are 16 possible second generation pedigrees, all equally likely. Therefore, each second generation pedigree has a probability of 1/16. The probability of the first generation NN and any of the second generation pedigrees is $\frac{1}{4}\left(\frac{1}{16}\right)=\frac{1}{64}$.

The probability that the first generation pedigree is NM is 1/4. There are 4 possible second generation pedigrees, all equally likely. Therefore, each second generation pedigree has a probability of 1/4. The probability of the first generation NM and any of the second generation pedigrees is $\frac{1}{4}\left(\frac{1}{4}\right)=\frac{1}{16}$.

The probability that the first generation pedigree is MN is 1/4. There are 4 possible second generation pedigrees, all equally likely. Therefore, each second generation pedigree has a probability of 1/4. The probability of the first generation MN and any of the second generation pedigrees is $\frac{1}{4}\left(\frac{1}{4}\right)=\frac{1}{16}$.

The probability that the first generation pedigree is MM is 1/4. There is only 1 possible second generation pedigree. Therefore, the second generation pedigree has a probability of 1. The probability of the first generation MM and the only second generation pedigrees is $\frac{1}{4}(1)=\frac{1}{4}$.

The probability that a single, normal cell that divides into two offspring will result in at least 1 mutant cell after the 2^{nd} generation is:

$$P(\text{at least one mutant cell after 2nd generation}) = 1 - P(\text{0 mutant cells after 2nd generation})$$

$$= 1 - P(NNNN) = 1 - \frac{1}{64} = 1 - .0156 = .9844$$

3.96 Define the following events:

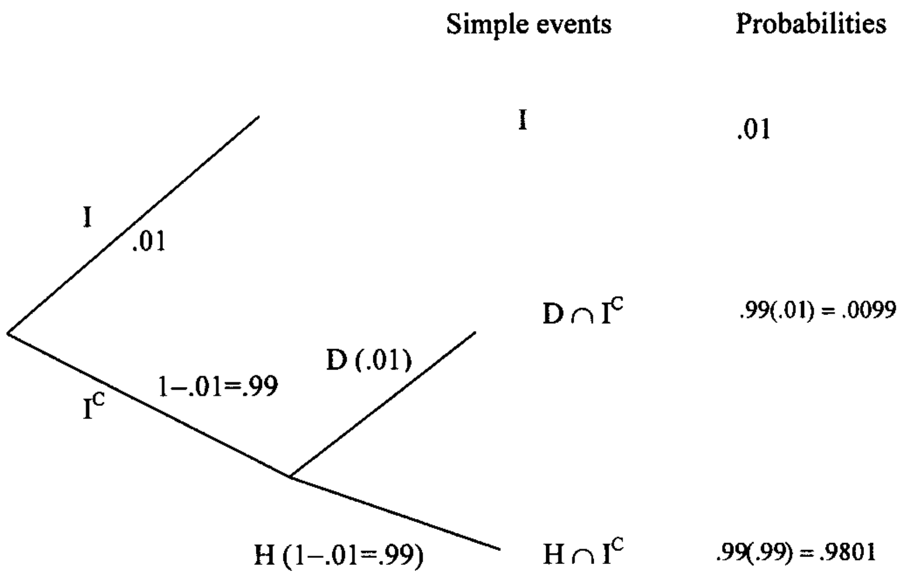
I: {Leak ignites immediately}

D: {leak has delayed ignition}

H: {gas cloud will harmlessly disperse}

From the problem, $P(I) = .01$, $P(D \mid I^c) = .01$

A probability tree is:



$P(I \cup D) = 1 - P(H) = 1 - .9801 = .0199$

3.98 If n elements are selected from a population in such a way that every set of n elements in the population has an equal probability of being selected, the n elements are said to be a random sample.

3.100 Random number generators automatically generate a random sample. They are available in table form and they are built into most statistical software packages.

3.102 a. Let the 10 elements be A, B, C, D, E, F, G, H, I, J. The number of ways to draw 2 elements is:

AB	AI	BH	CH	DI	FG	HI
AC	AJ	BI	CI	DJ	FH	HJ
AD	BC	BJ	CJ	EF	FI	IJ
AE	BD	CD	DE	EG	FJ	
AF	BE	CE	DF	EH	GH	
AG	BF	CF	DG	EI	GI	
AH	BG	CG	DH	EJ	GJ	

Using combinatorial mathematics, the number of ways to select a sample of $n = 2$ elements from 10 is:

$$\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10 \cdot 9 \cdot 8 \cdots 2 \cdot 1}{2 \cdot 1 \cdot 8 \cdot 7 \cdots 2 \cdot 1} = 45$$

b. Since there are 45 possibilities, the probability of selecting any particular sample is 1/45 = .022.

c. Answers will vary. Suppose we number our 10 elements from 0 to 9. Using Table I, Appendix A, start in column 8, row 36, last digit, going down. The first 2 numbers are 0 and 2. The next 2 numbers are 7 and 7. Since we cannot select 7 twice, we go to the next number. It, too, is a 7. We then go to the next number, which is 0. Thus, our second sample is 7 and 0. Continuing in this manner, our 20 samples will be:

(0, 2) (7, 0) (6, 2) (8, 9) (5,1) (8,2) (0,2) (3,5) (2,0) (4,2) (6,1) (8,3) (5,2) (7,0) (7,4) (3,6) (8,2) (3,1) (2,8) (9,2)

There are several samples that are the same: There are 3 samples of (0, 2). There are also 2 samples of both (0,7) and (2,8).

Since there are only 45 different samples and we are choosing 20, it would not be unusual to have the same sample selected more than once.

The probability of getting at least one match in 20 samples is 1 minus the probability of getting no matches. Since we have 45 different possibilities to choose from, each time I take a sample, I have 45 things to choose from. Thus, in 20 trials, I would have 45^{20} ways to select the samples. In order to get no matches in the 20 trials, I would only have $P_{20}^{45} = \dfrac{45!}{(45-20)!} = \dfrac{45!}{25!}$ ways to select the 20 samples. Thus,

$$P(\text{no matches}) = \frac{P_{20}^{45}}{45^{20}} = \frac{\frac{45!}{25!}}{45^{20}} = .006 \text{ and}$$

$$P(\text{at least 1 match}) = 1 - P(\text{no matches}) = 1 - .0067 = .9933\,.$$

3.104 Using MINITAB, click on **calc** in the menu, then **Random data**, and **Uniform**. In the screen that appears, enter **10** in the **Generate rows of data** blank, and enter **C1** in the **Store in column** blank. Enter **1** in the **Lower endpoint** blank and **200000** in the **Upper endpoint** blank. Then click **OK**. The ten randomly generated numbers between 1 and 200,000 will appear in column 1. One example of the data generated is:

43398
170853
77774
128675
62539
120106
82741
187531
123659
154069

3.106 First, number the households from 1 to 534,322. Using the random number table, select a starting point that consists of 6 digits. Following either the row or column, select successive 6 digit numbers between 1 and 534,322 until 1000 different 6 digit numbers have been selected. The sample will consist of the 1000 households corresponding to the 1000 different numbers.

3.108 To create groups that are as close to equal as possible, we will make groups of 17, 17 and 16. First, we will number the rats from 1 to 50. Then we will generate 34 random numbers. The first 17 generated numbers will be assigned to the typical American diet. The next 17 generated numbers will be assigned to the Atkins-type diet. The remaining 16 rats will be assigned to rat chow.

The answers can vary. One possible solution follows. Using MINITAB, we will generate 50 random numbers between 1 and 50. After deleting all the duplicate numbers, there were 35 random numbers remaining. The first 34 numbers were kept. The first 17 numbers sorted were:

1 5 7 8 11 13 17 18 24 25 27 33 35 36 40 46 47

Rats assigned to the above numbers were placed in the American diet group.

The next 17 numbers sorted were:

2 4 6 12 14 20 23 26 28 29 30 31 37 38 39 42 44

Rats assigned to these numbers were placed in the Atkins-type diet. All of the remaining rats were assigned to the rat chow diet.

3.110 a. The probability that account 3,241 is chosen is 1 out of 5,382 or 1/5,382 = .000186

b. Using MINITAB, click on **calc** in the menu, then **Random data**, and **Uniform**. In the screen that appears, enter **10** in the **Generate rows of data** blank, and enter **C1** in the **Store in column** blank. Enter **1** in the **Lower endpoint** blank and **5382** in the **Upper endpoint** blank. Then click **OK**. The ten randomly generated numbers between 1 and 5,382 will appear in column 1. One example of the data generated is:

904
4780
5095
4693
2077
508
3002
2928
3584
832

c. No. If we are looking at the exact numbers chosen, both samples are equally likely. The total number of ways one can draw a sample of size 10 from a population of 5382 is a combination of 5382 items taken 10 at a time or:

$$\binom{5280}{10} = 1.036\text{x}10^{27}$$

The probability of getting either of these two exact samples is $\frac{1}{1.036 \times 10^{27}}$.

3.112 Suppose you are ordering a new car. You have several choices to make:

Number of doors – either 2 or 4
Type of transmission – automatic or manual
Color – Tan, Blue, Red, White, or Black
Type of interior – Leather, Upgraded cloth, or Regular cloth

How many different combinations of cars are possible?

We can apply the multiplicative rule to find the number of combinations. There are 2 choices for number of doors, 2 choices for type of transmission, 4 choices of color, and 3 choices for type of interior. The total number of combinations is 2 x 2 x 4 x 3 = 48.

3.114 Suppose you have a total of 12 students assigned to a set crew for a school play. Three need to be assigned to the stage crew, four need to be assigned to the costume crew, and five need to be assigned to the design crew. How many ways can the 12 students be assigned to the 3 jobs?

We can use the Partitions Rule to obtain the total number of ways:

$$\frac{N!}{n_1!n_2!n_3!} = \frac{12!}{3!4!5!} = \frac{12\cdot 11\cdot 10\cdots 1}{3\cdot 2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 27,720$$

3.116 a. $\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = 20$

b. $P_2^5 = \frac{5!}{(5-2)!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1} = 20$

c. $P_2^4 = \frac{4!}{(4-2)!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = 12$

d. $\binom{100}{98} = \frac{100!}{98!(100-98)!} = \frac{100\cdot 99\cdot 98\cdot \cdots \cdot 1}{98\cdot 97\cdot 96\cdot \cdots \cdot 1\cdot 2\cdot 1} = \frac{100\cdot 99}{2} = 4950$

e. $\binom{50}{50} = \frac{50!}{50!(50-50!)} = 1$

f. $\binom{50}{0} = \frac{50!}{0!(50-0!)} = 1$

g. $P_3^5 = \frac{5!}{(5-3)!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = 60$

h. $P_0^{10} = \frac{10!}{(10-0)!} = 1$

3.118 a. 6

b. Using the multiplicative rule, the number of simple events is 6(6) = 36.

c. Using the multiplicative rule, the number of simple events is 6(6)(6)(6) = 1296.

d. Using the multiplicative rule, the number of simple events is $\underbrace{6(6)\cdots 6}_{n} = 6^n$.

3.120 a. We can use the multiplicative rule for this exercise. For this exercise, there are 7 groups with the following number of elements in each group:

Unknown – 5	Slight – 4	Moderate – 3
Unworn – 2	Light-Moderate – 2	Moderate-heavy – 1
Heavy – 1		

Using the multiplicative rule, there would be 5(4)(3)(2)(2)(1)(1) = 240 different samples possible.

b. Without using the "unknown" group, there would be (4)(3)(2)(2)(1)(1) = 48 different samples possible.

3.122 a. The number of electrode pairs on the ankle is a combination of 6 things taken 2 at a time or $\binom{6}{2} = \frac{6!}{2!4!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1} = 15$

b. The number of electrode pairs on the knee is a combination of 10 things taken 2 at a time or $\binom{10}{2} = \frac{10!}{2!8!} = \frac{10\cdot 9\cdot 8\cdots 1}{2\cdot 1\cdot 8\cdot 7\cdot 6\cdots 1} = 45$

c. The number of possible electrode pairs where one electrode is attached to the knee and the other is attached to the ankle is 10(6) = 60.

3.124 To solve each of these parts, we think of the teams being formed in sequence. First we determine how many ways we can select employees for the first team. Then, from the remaining employees, how many ways can we select from them for the second team. Then the third team would consist of the remaining, unselected employees. The multiplicative rule would then be used to determine the total number of ways these sets of three teams can be selected.

a. $\frac{10!}{3!3!4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1} = 4200$

b. $\frac{10!}{2!3!5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 2520$

c. $\frac{10!}{1!4!5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{1\cdot 4\cdot 3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 1260$

d. $\frac{10!}{2!4!4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 4\cdot 3\cdot 2\cdot 1} = 3150$

3.126 Using the multiplicative rule, there would be 10(10)(10)(10)(10)(10)(10)(10)(10) = 10^9 = 1,000,000,000.

The first three digits of a Chicago zip code is 606. The total number of available zip codes for Chicago would be 10(10)(10)(10)(10)(10) = 10^6 = 1,000,000.

3.128 a. Since order matters, the number of ways to order the 8 networks over the eight days is 8(7)(6)(5)(4)(3)(2)(1) = 40,320.

b. The probability that ESPN is selected on Monday, July 11th is 1/8. The probability that any network is selected on any particular day is 1/8.

c. Since there are 2 Sundays, the probability that any one network is selected on a Sunday is 2/8 = 1/4. Thus, the probability that MTV is selected on a Sunday is 1/4.

3.130 a. For the number 3, there are 3 partitions: 3, 2+1, and 1+1+1. Now, we must determine how many ways we can put the color/shape combinations with these partitions.

For the partition 3, there are 4 ways to put the color/shape combinations:
3♥, 3♦, 3♠, and 3♣

For the partition 2+1, there are 12 ways to put the color/shape combinations:
2♥+ 1♦, 2♥+ 1♠, 2♥+ 1♣,
2♦+ 1♥, 2♦+ 1♠, 2♦+ 1♣,
2♠+ 1♥, 2♠+ 1♦, 2♠+ 1♣,
2♣+ 1♥, 2♣+ 1♦, 2♣+ 1♠.

For the partition 1+1+1, there are 4 ways to put the color/shape combinations:
1♥+ 1♦+ 1♠, 1♥+ 1♦+ 1♣, 1♥+ 1♠+ 1♣, 2♦+ 1♠+ 1♣.

Thus, the total number of colored partitions of the number 3 is 4 + 12 + 4 = 20.

b. For the number 5, there are 7 partitions: 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, and 1+1+1+1+1. Now, we must determine how many ways we can put the color/shape combinations with these partitions.

For the partition 5, there are 4 ways to put the color/shape combinations:
5♥, 5♦, 5♠, and 5♣

For the partition 4+1, there are 12 ways to put the color/shape combinations:
4♥+ 1♦, 4♥+ 1♠, 4♥+ 1♣,
4♦+ 1♥, 4♦+ 1♠, 4♦+ 1♣,
4♠+ 1♥, 4♠+ 1♦, 4♠+ 1♣,
4♣+ 1♥, 4♣+ 1♦, 4♣+ 1♠.

For the partition 3+2, there are 12 ways to put the color/shape combinations:
3♥+ 2♦, 3♥+ 2♠, 3♥+ 2♣,
3♦+ 2♥, 3♦+ 2♠, 3♦+ 2♣,
3♠+ 2♥, 3♠+ 2♦, 3♠+ 2♣,
3♣+ 2♥, 3♣+ 2♦, 3♣+ 2♠.

For the partition 3+1+1, there are 12 ways to put the color/shape combinations:
3♥+ 1♦+ 1♠, 3♥+ 1♦+ 1♣, 3♥+ 1♠ + 1♣,
3♦+ 1♥+ 1♠, 3♦+ 1♥+ 1♣, 3♦+ 1♠ + 1♣,
3♠+ 1♥+ 1♦, 3♠+ 1♥+ 1♣, 3♠+ 1♦ + 1♣,
3♣+ 1♥+ 1♦, 3♣+ 1♥+ 1♠, 3♣+ 1♦+ 1♠.

For the partition 2+2+1, there are 12 ways to put the color/shape combinations:
2♥+ 2♦+ 1♠, 2♥+ 2♦+ 1♣, 2♥+ 2♠ + 1♦, 2♥+ 2♠ + 1♣, 2♥+ 2♣+ 1♦,
2♥+ 2♣+ 1♠, 2♦+ 2♠ + 1♥, 2♦+ 2♠ + 1♣, 2♦+ 2♣+ 1♥, 2♦+ 2♣+ 1♠,
2♠+ 2♣+ 1♥, 2♠+ 2♣ + 1♦,

For the partition 2+1+1+1, there are 4 ways to put the color/shape combinations:
2♥+ 1♦+ 1♠+ 1♣,
2♦+ 1♥+ 1♠ + 1♣,
2♠+ 1♥+ 1♦ + 1♣,
2♣+ 1♥+ 1♦+ 1♠.

For the partition 1+1+1+1+1, there are 0 ways to put the color/shape combinations.

Thus the total number of colored partitions of the number 5 is

4 + 12 + 12 + 12 + 12 + 4 = 56.

3.132 a. Using the multiplicative rule, there would be 26(26)(26)(10)(10)(10) = 17,576,000 different license plates.

b. By reversing the procedure, there would be 10(10)(10)(26)(26)(26) = 17,576,000 new tags.

c. The total number of licenses available would be 2(17,576,000) = 35,152,000.

3.134 Since there are $n_1 = 5$ choices of style, $n_2 = 8$ choices of color, and $n_3 = 2$ choices of transmission, there would be

$$n_1 n_2 n_3 = (5)(8)(2) = 80$$

different automobiles the dealer would need to stock.

3.136 The six players can be in any of six distinct positions. By the permutation rule, this can be accomplished in

$P_6^6 = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = 720$ ways. Since there is only one serving position, the probability that the player is the best server is 1/6.

3.138 $P(A \mid B)$ is the probability that event A occurs given that event B has already occurred.

$P(B \mid A)$ is the probability that event B occurs given that event A has already occurred.

3.140 If events A and B are mutually exclusive, then if one event occurs, the other event cannot occur. If events A and B are mutually exclusive, then $P(B \mid A) = 0$.

3.142 First, we find the following probabilities:

$$P(A \cap B_1) = P(A \mid B_1)P(B_1) = .4(.2) = .08$$
$$P(A \cap B_2) = P(A \mid B_2)P(B_2) = .25(.15) = .0375$$
$$P(A \cap B_3) = P(A \mid B_3)P(B_3) = .6(.65) = .39$$
$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) = .08 + .0375 + .39 = .5075$$

a. $P(B_1 \mid A) = \dfrac{P(A \cap B_1)}{P(A)} = \dfrac{.08}{.5075} = .158$

b. $P(B_2 \mid A) = \dfrac{P(A \cap B_2)}{P(A)} = \dfrac{.0375}{.5075} = .074$

c. $P(B_3 \mid A) = \dfrac{P(A \cap B_3)}{P(A)} = \dfrac{.39}{.5075} = .768$

3.144 a. $P(G_1 \mid D) = \dfrac{P(G_1 \cap D)}{P(D)} = \dfrac{P(D \mid G_1)P(G_1)}{P(D)} = \dfrac{.5(.2)}{.34} = .294$

b. $P(G_2 \mid D) = \dfrac{P(G_2 \cap D)}{P(D)} = \dfrac{P(D \mid G_2)P(G_2)}{P(D)} = \dfrac{.3(.8)}{.34} = .706$

3.146 a. Converting the percentages to probabilities,

$P(275 - 300) = .52$, $P(305 - 325) = .39$, and $P(330 - 350) = .09$.

b. Using Bayes Theorem,

$$P(275-300 \mid CC) = \frac{P(275-300 \cap CC)}{P(CC)}$$

$$= \frac{P(CC \mid 275-300)P(275-300)}{P(CC \mid 275-300)P(275-300) + P(CC \mid 305-325)P(305-325) + P(CC \mid 330-350)P(330-350)}$$

$$= \frac{.775(.52)}{.775(.52) + .77(.39) + .86(.09)} = \frac{.403}{.403 + .3003 + .0774} = \frac{.403}{.7807} = .516$$

3.148 a. Define the following events:
H: {person has HIV}
P: {positive test}
N: {Negative test}

From the exercise, we know $P(H) = .008$, $P(P \mid H) = .99$, $P(N \mid H^c) = .99$. Using Bayes' Rule, the probability is:

$$P(H \mid P) = \frac{P(H \cap P)}{P(P)} = \frac{P(H)P(P \mid H)}{P(H)P(P \mid H) + P(H^c)P(P \mid H^c)}$$

$$= \frac{.008(.99)}{.008(.99) + .992(.01)} = \frac{.00792}{.00792 + .00992} = \frac{.00792}{.01784} = .44395$$

b. In East Asia, $P(H) = .001$. Using Bayes' Rule, the probability is:

$$P(H \mid P) = \frac{P(H \cap P)}{P(P)} = \frac{P(H)P(P \mid H)}{P(H)P(P \mid H) + P(H^c)P(P \mid H^c)}$$

$$= \frac{.001(.99)}{.001(.99) + .999(.01)} = \frac{.00099}{.00099 + .00999} = \frac{.00099}{.01098} = .09016$$

c. Since the tests are independent, we know:

$P(H \mid P \text{ on first}) = .09016$ and $P(H \mid P \text{ on second}) = .09016$.

The probability that the person has HIV given both tests are positive is
$\left[P(H \mid P \text{ on first})\right]\left[P(H \mid P \text{ on second})\right] = .44395(.44395) = .19709$

d. In East Asia, the probability is
$\left[P(H \mid P \text{ on first})\right]\left[P(H \mid P \text{ on second})\right] = .09016(.09016) = .00813$

3.150 Define the following events:

H: {NDE detects a "hit"}

D: {Defect exists}

From the problem, we know $P(H \mid D) = .97$, $P(H \mid D^c) = .005$, and $P(D) = 1/100 = .01$.

In order to find $P(D \mid H)$ we must first fine $P(H)$.

$$P(H) = P(H \cap D) + P(H \cap D^c) = P(H \mid D)P(D) + P(H \mid D^c)P(D^c) = .97(.01) + .005(1 - .01)$$
$$= .0097 + .00495 = .01465$$

$$P(D \mid H) = \frac{P(D \cap H)}{P(H)} = \frac{P(H \mid D)P(D)}{P(H)} = \frac{.97(.01)}{.01465} = .6621$$

3.152 Define the following events:

S: {System shuts down}
F_1: {Hardware failure}
F_2: {Software failure}
F_3: {Power failure}

From the Exercise, we know:

$P(F_1) = .01$, $P(F_2) = .05$, and $P(F_3) = .02$. Also, $P(S \mid F_1) = .73$, $P(S \mid F_2) = .12$, and $P(S \mid F_3) = .88$.

The probability that the current shutdown is due to a hardware failure is:

$$P(F_1 \mid S) = \frac{P(F_1 \cap S)}{P(S)} = \frac{P(S \mid F_1)P(F_1)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.73(.01)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.0073}{.0073 + .006 + .0176} = \frac{.0073}{.0309} = .2362$$

The probability that the current shutdown is due to a software failure is:

$$P(F_2 \mid S) = \frac{P(F_2 \cap S)}{P(S)} = \frac{P(S \mid F_2)P(F_2)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.12(.05)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.006}{.0073 + .006 + .0176} = \frac{.006}{.0309} = .1942$$

The probability that the current shutdown is due to a power failure is:

$$P(F_3 \mid S) = \frac{P(F_3 \cap S)}{P(S)} = \frac{P(S \mid F_3)P(F_3)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.88(.02)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.0176}{.0073 + .006 + .0176} = \frac{.0176}{.0309} = .5696$$

3.154 a. The events A and B are not mutually exclusive. Event A (Tampa Bay Rays win the World Series next year) and Event B (Evan Longoria, Rays Infielder, hits 75 home runs next year) could both happen next year. Since both events could happen at the same time, they cannot be mutually exclusive.

b. The events A and B are not mutually exclusive. If event B (Psychiatric patient Tony has the fastest stimulus response time of 2.3 seconds) occurred, then event A (Psychiatric patient Tony responds within 5 seconds) also occurred. Thus events A and B are not mutually exclusive.

c. Events A and B are mutually exclusive. If event A (High school graduate Cindy enrolls at the University of South Florida next year) occurs, then event B (High school graduate Cindy does not enroll in college next year) cannot occur.

3.156 a. The first rule of probability is that all probabilities of simple events must be between 0 and 1. In this problem, all the probabilities are between 0 and 1. The second rule of probability is that the sum of all probabilities associated with the simple events sum to 1. In this problem $P(S_1)+P(S_2)+P(S_3)+P(S_4)=.2+.1+.3+.4=1.0$.

b. $P(A)=P(S_1)+P(S_4)=.2+.4=.6$

3.158 $P(A\cup B)=P(A)+P(B)-P(A\cap B)=.7+.5-.4=.8$

3.160 a. $P(A\cap B)=0$

$P(B\cap C)=P(2)=.2$

$P(A\cup C)=P(1)+P(2)+P(3)+P(5)+P(6)=.3+.2+.1+.1+.2=.9$

$P(A\cup B\cup C)=P(1)+P(2)+P(3)+P(4)+P(5)+P(6)=.3+.2+.1+.1+.1+.2=1.0$

$P(B^c)=P(1)+P(3)+P(5)+P(6)=.3+.1+.1+.2=.7$

$P(A^c\cap B)=P(2)+P(4)=.2+.1=.3$

$$P(B\,|\,C)=\frac{P(B\cap C)}{P(C)}=\frac{P(2)}{P(2)+P(5)+P(6)}=\frac{.2}{.2+.1+.2}=\frac{.2}{.5}=.4$$

$$P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{0}{P(A)}=0$$

b. Since $P(A\cap B)=0$, and $P(A)P(B)>0$, these two would not be equal, implying *A* and *B* are not independent. However, *A* and *B* are mutually exclusive, since $P(A\cap B)=0$.

c. $P(B)=P(2)+P(4)=.2+.1=.3$. But $P(B\,|\,C)$, calculated above, is .4. Since these are not equal, *B* and *C* are not independent. Since $P(B\cap C)=.2$, *B* and *C* are not mutually exclusive.

3.162 a. *P*(Red) = *P*(Red from Urn 1) + *P*(Red from Urn 2)

$$= P(\text{Red}\,|\,\text{Urn 1})\cdot P(\text{Urn 1}) + P(\text{Red}\,|\,\text{Urn 2})\cdot P(\text{Urn 2})$$

$$=\frac{4}{10}\cdot\frac{1}{6}+\frac{6}{10}\cdot\frac{5}{6}=\frac{34}{60}\approx .567$$

b. $$P(\text{Urn 1}\,|\,\text{Red})=\frac{P(\text{Urn 1 and Red})}{P(\text{Red})}=\frac{P(\text{Red}\,|\,\text{Urn 1})\cdot P(\text{Urn 1})}{P(\text{Red})}$$

$$=\frac{\frac{4}{10}\cdot\frac{1}{6}}{\frac{34}{60}}=\frac{4}{34}\approx .118$$

3.164 a. $6! = 6(5)(4)(3)(2)(1) = 720$

b. $\binom{10}{9} = \frac{10!}{9!(10-9)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cdots \cdot 1}{9 \cdot 8 \cdot 7 \cdot \cdots \cdot 1 \cdot 1} = 10$

c. $\binom{10}{1} = \frac{10!}{1!(10-1)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cdots \cdot 1}{1 \cdot 9 \cdot 8 \cdot \cdots \cdot 1} = 10$

d. $P_2^6 = \frac{6!}{(6-2)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 30$

e. $\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 20$

f. $0! = 1$

g. $P_4^{10} = \frac{10!}{(10-4)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cdots \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5040$

h. $P_2^{50} = \frac{50!}{(50-2)!} = \frac{50 \cdot 49 \cdot 48 \cdot \cdots \cdot 1}{48 \cdot 47 \cdot \cdots \cdot 1} = 2450$

3.166 Define the following events:

A: {Couple has no child by choice}
B: {Couple has no child because they are biologically infertile}
C: {Couple is sterile}

From the problem, we know $P(C) = P(A \cup B) = .06$. Also, we know $P(B \mid C) = .64$.

The event that a couple is both sterile and infertile is $C \cap B$.

$$P(C \cap B) = P(B \mid C)P(C) = .64(.06) = .0384.$$

3.168 a. $\binom{50}{5} = \frac{50!}{5!45!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2{,}118{,}760$

b. Answers will vary. First, we number the students from 1 to 50. We will use Table I, Appendix A to select the sample of size 5. Suppose we start in column 13, row 20, the first 2 digits, going down. The first 2 digit number selected is 25. The next number is 99. Since 50 is the largest number we can select, we ignore 99 and go to the next number, which is 96. Again, we skip this number and go to the next which is 18. The next number is 36, followed by 50. We ignore the next 3 numbers, 79, 80, and 96. We then select the next number which is 34. Thus, the 5 numbers selected for our sample are 25, 18, 36, 50, and 34.

3.170 From the output for Exercise 2.189, we see that there are 36 transects that are not contaminated with oil and 60 that are. Thus, *P*(transect contaminated with oil)

= 60 / 96 = .625.

3.172 Define the following event:

A_i : {Patient i uses herbal medicine against doctor's advice}

From problem, $P(A_i) = .51$.

a. $P(A_1) = .51$

b. If two events, A_1 and A_2, are independent, then $P(A_1 \cap A_2) = P(A_1)P(A_2)$.

Thus, $P(A_1 \cap A_2) = P(A_1)P(A_2) = .51(.51) = .2601$.

c.
$$P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = P(A_1)P(A_2)P(A_3)P(A_4)P(A_5) = .51(.51)(.51)(.51)(.51) = .0345.$$

d. No. Since the probability of all 5 independently selected patients using herbal medicines against their doctor's advice is so small, we would not expect this to happen.

3.174 a. Let *S* denote the event that an insect travels toward the pheromone and *F* the event that an insect travels toward the control. The sample space associated with the experiment of releasing five insects has 32 simple events:

SSSSS	*FSSFS*	*SSFFF*	*FFSFS*
SSSSF	*FSSSF*	*SFSFF*	*FFFSS*
SSSFS	*SFFSS*	*SFFSF*	*FFFFS*
SSFSS	*SFSFS*	*SFFFS*	*FFFSF*
SFSSS	*SFSSF*	*FSSFF*	*FFSFF*
FSSSS	*SSFFS*	*FSFSF*	*FSFFF*
FFSSS	*SSFSF*	*FSFFS*	*SFFFF*
FSFSS	*SSSFF*	*FFSSF*	*FFFFF*

If the pheromone under study has no effect, then each simple event is equally likely and occurs with probability 1/32.

b. The probability that all five insects travel toward the pheromone is *P*(*SSSSS*) = 1/32 = .03125.

c. The probability that exactly four of the five insects travel toward the pheromone is:

P(*SSSSF*) + *P*(*SSSFS*) + *P*(*SSFSS*) + *P*(*SFSSS*) + *P*(*FSSSS*) = 5/32 = .15625

d. Since the probability that exactly four of the five insects travel toward the pheromone is not small (.15625), this is not an unusual event. (This probability was computed assuming the pheromone has no effect.)

3.176 First we will define some events:

A: {Player is white}

B: {Player is black}

C: {Player is a guard}

D: {Player is a forward}

E: {Player is a center}

a. P(Player is white) = $P(A) = \frac{84}{368} = .228$

b. P(Player is a center) = $P(E) = \frac{62}{368} = .168$

c. P(Player is African-American and a guard) = $P(B \cap C) = \frac{128}{368} = .348$

d. P(Player is not a guard) = $P(C^c) = 1 - P(C) = 1 - \frac{154}{368} = 1 - .418 = .582$

e. P(Player is white or a center) $= P(A \cup E) = P(A) + P(E) - P(A \cap E$

$$= \frac{84}{368} + \frac{62}{368} - \frac{28}{368} = .228 + .168 - .076 = .320.$$

f. $$P(E \mid A) = \frac{P(E \cap A)}{P(A)} = \frac{\frac{28}{368}}{\frac{84}{368}} = \frac{28}{84} = .333$$

g. $$P(E \mid B) = \frac{P(E \cap B)}{P(B)} = \frac{\frac{34}{368}}{\frac{284}{368}} = \frac{34}{284} = .120$$

h. Events A and E are independent if $P(E \mid A) = P(E)$. From part f, $P(E \mid A)$ = .333.

$$P(E) = \frac{62}{368} = .168$$

Since $P(E \mid A) \neq P(E)$, the events are not independent.

i. Since the events {White player} and {Center} are not independent, this supports the theory of "stacking". Also, from part f, we found that $P(E \mid A) = .333$ and from part g,

$P(E \mid B) = .120$. Thus, the probability of playing Center depends on race. Again, this supports the theory of "stacking".

3.178 Define the following events:

A: {Wheelchair user had an injurious fall}
B: {Wheelchair user had all 5 features}
C: {Wheelchair user had at least 1 feature but not all 5 features}
D: {Wheelchair user had no features}

a. $P(A) = \frac{48}{306} = .157$

b. $P(B) = \frac{9}{306} = .029$

c. $P(A^c \cap D) = \frac{89}{306} = .291$

d. $P(A \mid D) = \frac{P(A \cap D)}{P(D)} = \frac{20/306}{109/306} = \frac{20}{109} = .183$

3.180 First, define the following event:

A: {CVSA correctly determines the veracity of a suspect} $P(A) = .98$ (from claim)

a. The event that the CVSA is correct for all four suspects is the event $A \cap A \cap A \cap A$.
$P(A \cap A \cap A \cap A) = .98(.98)(.98)(.98) = .9224$

b. The event that the CVSA is incorrect for at least one of the four suspects is the event $(A \cap A \cap A \cap A)^c$.
$P(A \cap A \cap A \cap A)^c = 1 - P(A \cap A \cap A \cap A) = 1 - .9224 = .0776$

c. In part b, we found the probability of at least 1 incorrect result was .0776. The probability of at least 2 incorrect results would be less than .0776. Thus, it would be very unlikely to see 2 incorrect results if the manufacturer's claim was correct. Thus, we would conclude that the accuracy was less than 98%.

3.182 a. The number of ways to select 5 members from 15 is a combination of 15 things taken 5 at a time, or

$$\binom{15}{5} = \frac{15!}{5!(15-5)!} = \frac{15 \cdot 14 \cdot 13 \cdot \cdots \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 10 \cdot 9 \cdot \cdots \cdot 1} = 3003$$

b. To get no democrats, all 5 members chosen must be Republicans. The number of ways to select 5 members from 8 is a combination of 8 things taken 5 at a time or

$$\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cdots \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 56$$

The probability that no Democrat is appointed is the number of ways to select no Democrats divided by the total number of ways to select 5 members, or

$$P(\text{no Democrats}) = \frac{56}{3003} = .0186$$

Since this probability is so small, we have either seen a rare event or the appointments were not made at random.

c. To get a majority of Republicans, 3, 4, or 5 members must be Republicans. We computed the number of ways to get 5 Republicans in part **b**. The number of ways to select 4 Republicans from 8 and 1 Democrat from 7 is:

$$\binom{8}{4}\binom{7}{1} = \frac{8!}{4!(8-4)!}\cdot\frac{7!}{1!(7-1)!} = \frac{8\cdot 7\cdot\cdots\cdot 1}{4\cdot 3\cdot 2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot\cdots\cdot 1}{1\cdot 6\cdot 5\cdot\cdots\cdot 1} = 490$$

The number of ways to select 3 Republicans from 8 and 2 Democrats from 7 is:

$$\binom{8}{3}\binom{7}{2} = \frac{8!}{3!(8-3)!}\cdot\frac{7!}{2!(7-2)!} = \frac{8\cdot 7\cdot\cdots\cdot 1}{3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot\cdots\cdot 1}{2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 1176$$

The probability that the majority of committee members are Republican is the number of ways to get 3, 4, or 5 Republicans divided by the total number of ways to get 5 members, or

$$P(\text{Majority Republican}) = \frac{56+490+1176}{3003} = \frac{1722}{3003} = .5734$$

This is not an unusual event because the probability is so high. There would be no reason to doubt the governor made the selections randomly.

3.184 Define the following events:

M: {electrical switching device can monitor the quality of the power running through it}

W: {Buyer does not wire switch for monitoring}

From the problem we know $P(M) = .9$ and $P(W \mid M) = .9$

$$P(M \text{ and } W^c) = P(M \cap W^c) = P(W^c \mid M)P(M) = (1-.9)(.9) = .09$$

3.186 a. The psychologist is asked to select $n = 10$ high-anxiety subjects from a group of $N = 20$ subjects. The number of simple events is given by the combinations rule:

$$\binom{N}{n} = \binom{20}{10} = \frac{20!}{10!10!} = 184,756$$

b. If the psychologist is guessing, then each of the simple events is equally likely and occurs with probability 1/184,756.

c. The probability that the psychologist guesses all classifications correctly is:

$$\frac{k}{184{,}756}$$

where *k* is the number of simple events in which all subjects are correctly classified.

To determine *k*, note that in order to make all classifications correctly, the psychologist must choose the 10 subjects from the 10 who are high-anxiety and must also choose 0 subjects from the 10 low-anxiety subjects. The number of ways this can be done is:

$$k = \binom{10}{10}\binom{10}{0} = \left(\frac{10!}{10!0!}\right)\left(\frac{10!}{0!10!}\right) = 1(1) = 1$$

Thus, the probability that the psychologist guesses all classifications correctly is only 1/184,756.

d. The probability that the psychologist guesses at least 9 of the 10 high-anxiety subjects correctly is:

$P(\text{At least 9 correct}) = P(\text{9 correct}) + P(\text{10 correct})$

In part **c**, we determined:

$P(\text{10 correct}) = 1/184{,}756$

Now, the probability of correctly identifying 9 high-anxiety subjects is:

$$P(\text{9 correct}) = \frac{c}{184{,}756}$$

where *c* is the number of simple events that result in 9 correct identifications of high-anxiety individuals.

In order to identify correctly 9 high-anxiety subjects, the psychologist must choose 9 of the 10 high-anxiety individuals and 1 of the 10 low-anxiety individuals. The number of ways this can be done is:

$$\binom{10}{9}\binom{10}{1} = \left(\frac{10!}{9!1!}\right)\left(\frac{10!}{1!9!}\right) = 10(10) = 100$$

Thus, $P(\text{9 correct}) = \dfrac{100}{184{,}756}$, and

$$P(\text{At least 9 correct}) = \frac{100}{184{,}756} + \frac{1}{184{,}756} = \frac{101}{184{,}756} \approx .0005$$

3.188 We can estimate the probabilities of the four combinations by using the relative frequency for each combination. The relative frequency is found by dividing the frequency by the total sample size of 4,208. These estimates are contained in the following table:

Sex Composition of First Two Children	Frequency	Probability
Boy-Boy	1,085	.2578
Boy-Girl	1,086	.2581
Girl-Boy	1,111	.2640
Girl-Girl	926	.2201
TOTAL	4,208	1.0000

If having boys or girls "runs in the family", then the probability of getting Boy-Boy or Girl-Girl should be greater than .5. From the above, an estimate of the probability of getting Boy-Boy or Girl-Girl is

$$P(\text{Boy}-\text{Boy or Girl}-\text{Girl}) = \frac{1{,}085+926}{4{,}208} = .478$$

Since this is less than .5, there is no evidence that having boys or girls "runs in the family".

3.190 a. The number of ways to draw 2 cards from 52 is:

$$\binom{52}{2} = \left(\frac{52!}{2!(52-2)!} = \frac{52\cdot 51\cdot 50\cdot \cdots \cdot 1}{2\cdot 1\cdot 50\cdot 49\cdot \cdots \cdot 1}\right) = 1326$$

In a deck of cards, there are 4 aces and 12 face cards. The number of ways to draw 1 ace and 1 face card is:

$$\binom{4}{1}\binom{12}{1} = \frac{4!}{1!(4-1)}\cdot\frac{12!}{1!(12-1)!} = \frac{4\cdot 3\cdot 2\cdot 1}{1\cdot 3\cdot 2\cdot 1}\cdot\frac{12\cdot 11\cdot 10\cdot \ldots \cdot 1}{1\cdot 11\cdot 10\cdot 9\cdot \ldots \cdot 1} = 4(12) = 48$$

Thus, the probability the dealer draws blackjack is $\frac{48}{1326} \approx .036$

b. For the player to win with blackjack, the player must draw blackjack while the dealer does not draw blackjack.

The probability the player draws blackjack is $\frac{48}{1326}$

Given the player draws blackjack, the number of ways the dealer can draw two cards is:

$$\binom{50}{2} = \frac{50!}{2!(50-2)!} = \frac{50\cdot 49\cdot 48\cdot \cdots \cdot 1}{2\cdot 1\cdot 48\cdot 47\cdot 46\cdot \cdots \cdot 1} = 1225$$

Given the player draws blackjack, the number of ways the dealer cannot draw blackjack is:

$$1225-\binom{3}{1}\binom{11}{1}=1225-\frac{3!}{1!(3-1)!}\cdot\frac{11!}{1!(11-1)!}=1225-\frac{3\cdot 2\cdot 1}{1\cdot 2\cdot 1}\cdot\frac{11\cdot 10\cdot 9\cdot ...\cdot 1}{1\cdot 10\cdot 9\cdot 8\cdot ...\cdot 1}$$
$$=1225-3(11)=1192$$

Thus, given the player draws blackjack, the probability the dealer will not draw blackjack is:

$$\frac{1192}{1225}\approx .973$$

The probability the player wins with blackjack is the probability the player draws blackjack and the dealer does not, which is:

$$\frac{48}{1326}\cdot\frac{1192}{1225}\approx .035$$

3.192 a. From the problem, $P(\text{Pass}) = .738$, $P(\text{Fail}) = .262$, $P(SLI \mid \text{Pass}) = .028$ and $P(SLI \mid \text{Fail}) = .205$.

b. $P(\text{Pass} \cap SLI) = P(SLI \mid \text{Pass})\, P(\text{Pass}) = .028\,(.738) = .0207$

$P(\text{Fail} \cap SLI) = P(\text{SLI} \mid \text{Fail})\, P(\text{Fail}) = .205\,(.262) = .0537$

We used the Multiplicative Rule of Probability.

c. $P(SLI) = P(\text{Pass} \cap SLI) + P(\text{Fail} \cap SLI) = .0207 + .0537 = .0744$.

We used the Probability of Union of Two Mutually Exclusive Events.

3.194 a. Refer to the simple events listed in Exercise 3.14.

$$P(\text{win on first roll}) = P(7 \text{ or } 11) = \frac{8}{36} = \frac{2}{9}$$

b. $P(\text{lose on first roll}) = P(\text{sum of 2 or 3}) = \dfrac{3}{36} = \dfrac{1}{12}$

c. If a player rolls a 4 on the first roll, the game will end on the next roll if:

1) the player rolls a 4
2) the player rolls a 7

$P(\text{game ends on } 2^{nd} \text{ roll} \mid 4 \text{ rolled on } 1^{st} \text{ roll}) = P(\text{roll 4or 7}) = (3 + 6)/36 = 9/36 = .25$.

3.196 If each of 3 players uses a fair coin, the sample space is:

HHH *HTT* *HHT* *THT* *HTH* *TTH* *THH* *TTT*

Since each event is equally likely, each event will have probability 1/8.

The probability of odd man out on the first roll is:

$P(HHT) + P(HTH) + P(THH) + P(HTT) + P(THT) + P(TTH)$
$= 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 6/8 = 3/4$

If one player uses a two-headed coin, the sample space will be (assume 3rd player has the two-headed coin):

HHH *THH* *HTH* *TTH*

Since each event is equally likely, each will have probability 1/4.

The probability the 3rd player is odd man out is $P(TTH) = 1/4$.

From the first part, the probability the 3rd player is odd man out is $P(HHT) + P(TTH)$ $= 1/8 + 1/8 = 1/4$. Thus, the two probabilities are the same.

3.198 Suppose we define the following event:

E: {Error produced when dividing}

From the problem, we know that $P(E) = 1/9{,}000{,}000{,}000$

The probability of no error produced when dividing is $P(E^c) = 1 - P(E)$ $= 1 - 1/9{,}000{,}000{,}000 = 8{,}999{,}999{,}999/9{,}000{,}000{,}000 = .999999999 \approx 1.0000$

Suppose we want to find the probability of no errors in 2 divisions (assuming each division is independent): $P(E^c \cap E^c) = .999999999(.999999999) = .999999999 \approx 1.0000$

Thus, in general, the probability of no errors in k divisions would be:

$$P(\underbrace{E^c \cap E^c \cap E^c \cap \cdots \cap E^c}_{k \text{ times}}) = P(E^c)^k = [8{,}999{,}999{,}999/9{,}000{,}000{,}000]^k$$

Suppose a user ran a program that performed 1 billion divisions. The probability of no errors in these 1 billion divisions would be:

$$P(E^c)^{1{,}000{,}000{,}000} = [8{,}999{,}999{,}999/9{,}000{,}000{,}000]^{1{,}000{,}000{,}000} = .9048$$

Thus, the probability of at least 1 error in 1 billion divisions would be

$$1 - P(E^c)^{1{,}000{,}000{,}000} = 1 - [8{,}999{,}999{,}999/9{,}000{,}000{,}000]^{1{,}000{,}000{,}000} = 1 - .9048 = .0952$$

For a heavy SAS user, this flawed chip would be a problem because the above probability is not that small.