"The Prisoner of Benda" is the 10th episode of the sixth season of the animated sitcom, *Futurama*. It aired on Comedy Central on August 19, 2010. In the episode, Professor Farnsworth and Amy build a machine that allows them to switch minds so that they may each pursue their lifelong dreams. However, they learn that the machine cannot be used twice on the same pairing of bodies. To try to return to their rightful bodies, they involve the rest of the crew in the mind switches, leaving each member free to pursue their own personal endeavors in a different crew member's body. The episode comprises multiple subplots.

The episode was written by Ken Keeler and directed by Stephen Sandoval and was met with positive reviews from critics. The issue of how each crew member can be restored to their correct body given the limitation of the switching device is solved in the episode by what David X. Cohen described in an interview as a mathematical theorem proved by Keeler, a Ph.D in Mathematics. The title and the story subplot is a reference to the 1894 adventure novel *The Prisoner of Zenda* by English novelist Anthony Hope. Series writer Eric Rogers considers this his favorite episode of the season.

**Plot**

Professor Farnsworth switches bodies with Amy using a new invention so that he can relive his youth. Likewise, Amy is nostalgic for her younger days of constantly eating and wishes to use the Professor's skinny body to gorge herself with food once again. Later, they find they cannot switch their bodies back with each other, because the device will not operate on the same pairing of bodies. The Professor thoughtlessly suggests they might be able to switch back to their original bodies with a third person. Bender switches bodies with the Professor (Amy's body) so he can perform a robbery without being identified. After realizing his mistake, the Professor, now in Bender's body, tires of trying to solve the problem. He decides to live a life of daring stunts and joins a robot circus.

Bender, now in Amy's body, goes aboard the yacht of Robo-Hungarian emperor Nikolai, planning to steal his crown. He knocks out Nikolai's first officer and cousin. After binding and gagging him, he realizes his timing is off and accidentally throws his watch through a metal detector, and is captured by the Emperor. When Bender states that he is really a robot who has switched bodies with a human, Nikolai reveals that he feels trapped by his wealth and wishes to live the life of a normal, "peasant" robot. Bender tricks him into switching bodies with a robot wash bucket and inhabits Nikolai's body, planning to live like an emperor. However, he discovers that Nikolai's fiancée and first officer are plotting to kill him, having been having an affair and planning to blame the burglar. They chase him to a meeting. Bender is saved with the assistance of the Professor, who is fired there in a cannon, and the circus' loyal Robo-Hungarian citizens.
Meanwhile, Leela switches bodies with Amy, thus inhabiting the Professor's body, when she comes to believe that Fry only loves her for her beauty. In order to beat Leela at her own game, Fry switches bodies with Zoidberg in an attempt to repulse Leela. This leads them to one-up each other using various disgusting acts while on a date, which climaxes when the two have sex with each other in their equally grotesque bodies, and reconcile. During this time, Amy has overeaten in Leela's body, making it overweight. She switches bodies with Hermes so she can continue eating while Hermes slims Leela's body back down. While eating, she witnesses Fry and Leela making out in the Professor and Dr. Zoidberg's bodies and loses her appetite for food. Concurrently, Zoidberg and Nikolai, in the respective bodies of Fry and the robot wash bucket, become friends and attempt to assume the lives of Fry and Bender, blowing up their apartment in the process. The bucket, now in Amy's body, professes its love to Scruffy, but he turns it away, noting their different lifestyles. Finally, two Globetrotters, Ethan "Bubblegum" Tate and "Sweet" Clyde Dixon, mathematically prove that everyone's minds can be restored using two additional bodies and then successfully do so, with themselves as the extras. The newly restored emperor makes Clyde duke, and in the last seconds of the episode, Bender realizes that the real crown is still in Nikolai's compartment. The credits start to roll, and Bender repeats his same line from the start, apparently wanting to re-steal the crown.

The theorem

In a 2010 interview, David X. Cohen revealed that the episode writer Ken Keeler, a PhD mathematician, penned and proved a theorem based on group theory, and then used it to explain the plot twist in this episode. However, Keeler does not feel it carries enough importance to be designated a theorem, and prefers to call it a proof. Cut-the-Knot, an educational math website created by Alexander Bogomolny, refers to Keeler's result as the "Futurama Theorem", while mathematician James Grime of the University of Cambridge calls it "Keeler's Theorem". In a 2012 interview, David X. Cohen said that this was probably the first time that a mathematical theorem was proved in a television script, and that it was probably Futurama's proudest mathematical moment.

The episode is based on a body swap scenario in which no pair of bodies can swap minds more than once. The proof demonstrates that after any sequence of mind switches, each mind can be returned to its original body by using only two additional individuals who had not yet swapped minds with anyone. A formal statement is as follows:

Let $A$ be a finite set, and let $x$ and $y$ be distinct objects that do not belong to $A$. Any permutation of $A$ can be reduced to the identity permutation by applying a sequence of distinct transpositions of $A \cup \{x,y\}$, each of which includes at least one of $x,y$.

The proof

The proof appears on the blackboard in the episode. The proof reduces to treating individual cycles separately, since all permutations can also be represented as products of disjoint cycles. So first let $\pi$ be some k-cycle on $[n]=\{1 \ldots n\}$. Without loss of generality, write:

$$\pi = \begin{pmatrix} 1 & 2 & \ldots & k & k+1 & \ldots & n \\ 2 & 3 & \ldots & 1 & k+1 & \ldots & n \end{pmatrix}.$$ 

Introduce the two new symbols $x$ and $y$, and write:

$$\pi^* = \begin{pmatrix} 1 & 2 & \ldots & k & k+1 & \ldots & n & x & y \\ 2 & 3 & \ldots & 1 & k+1 & \ldots & n & x & y \end{pmatrix}.$$ 

Let $(a b)$ be the transposition that interchanges $a$ and $b$. For any $i \in \{1 \ldots k-1\}$ let $\sigma$ be the permutation obtained as the (left to right) composition:

$$\sigma = (x \ 1)(x \ 2)\cdots(x \ i)(y \ i+1)(y \ i+2)\cdots(y \ k)(x \ i+1)(y \ 1).$$

Note that these are distinct transpositions, each of which exchanges an element of $[n]$ with one of $x,y$. By routine verification:
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\[ \pi^* \sigma = \left( \begin{array}{ccc} 1 & 2 & \ldots & n \ \\ x & y & \ldots & n \ \\ 1 & 2 & \ldots & x \end{array} \right) \]

That is, \( \sigma \) reverts the original k-cycle to the identity and leaves \( x \) and \( y \) switched (without performing \((xy)\)).

Next, let \( \pi \) be an arbitrary permutation on \([n]\). It consists of disjoint (nontrivial) cycles and each can be inverted as above in sequence after which \( x \) and \( y \) can be switched if necessary via \((xy)\), as was desired.

Cultural references

The episode's title and the Robo-Hungarian emperor subplot are references to the 1894 adventure novel *The Prisoner of Zenda*, in which a king is replaced by a commoner, by English novelist Anthony Hope.\(^7\) The episode also references the Turing test,\(^8\) a philosophical test designed to assess whether a computer can demonstrate true intelligence; actor Nicolas Cage;\(^9\) and the television character Fat Albert.\(^10\)

Broadcast and reception

"The Prisoner of Benda" originally aired on Comedy Central on August 19, 2010.\(^11\) In its original American broadcast, "The Prisoner of Benda" received 1,774 million viewers, down nearly 150,000 from the previous week's episode, "A Clockwork Origin". It received a 1.2 rating/2% share in the Nielsen ratings and a .8 rating/2% share in the 18–49 demographic, down two tenths of a point from the previous week.\(^11\)

The episode was unanimously well received by critics, who largely praised the episode's complexity and writing.\(^8\)\(^9\)\(^10\)\(^12\)\(^13\)\(^14\) Ken Keeler won a 2011 Writer's Guild Award for writing "The Prisoner of Benda."\(^16\)

Zack Handlen of The A.V. Club gave the episode a positive review, rating it an A−. Handlen compared the episode favorably to the season 4 episode "The Farnsworth Parabox", writing, "Really, this is pure silly from beginning to end, using the show's internal logic to arrive in unpredictable places, and bringing to life a piece of fan fiction I doubt anyone has ever had the courage to write. It reminded me more than a little of 'The Farnsworth Parabox': a premise that uses the whole cast to just the right amount." Merrill Barr of Film School Rejects also gave the episode a positive review, praising the character development of Scruffy the Janitor in particular. With respect to the logic puzzle nature of the plot,\(^8\) Barr stated, "I have never understood less of what was going on in a single episode of Futurama. And I couldn't have loved it more."\(^12\)

Sean Gandert of Paste rated the episode 9.1/10.\(^9\) He praised the intricate plot and writing, also comparing it to "The Farnsworth Parabox" and "Teenage Mutant Leela's Hurdles", writing, "...this sort of intertwined, Altman-esque plot offers up a level of complexity rarely offered by sitcoms, or any episodic television. 'The Prisoner of Benda' was a perfect example of how Futurama takes an extremely simple concept and executes it in a daring and unique way. It's not as showy as 'A Clockwork Origin' or 'The Late Philip J. Fry' from an animation standpoint, but with writing this tight we could be watching stick figures at 8 frames a second and it would still be worth seeing."\(^19\)

Robert Canning of IGN gave the episode a 9.5/10, stating "'The Prisoner of Benda' was an absolutely stellar episode. The high-concept idea of switching minds/bodies was taking [sic] to extremes and delivered fantastic lines and great laughs throughout."\(^14\) In response to this episode UGO Entertainment critic Alex Zalben concluded that "this season of Futurama, despite a miss or two, has hit what might possibly be a series high for the past few episodes, and I can't wait to watch the rest. Kudos to Comedy Central for bringing back what is now officially the best animated show on TV."\(^10\)

Alasdair Wilkins of io9 also praised the episode's mathematical complexity, saying "This episode is maybe the most epically overstuffed Futurama episode ever, with only the intersecting ensemble piece 'Three Hundred Big Boys' even coming close. 'The Prisoner of Benda' easily could have stretched out to a 70-minute DVD movie, but I'm just as happy to see it as 22 minutes of finely concentrated crazy."\(^15\) In an interview, *Futurama* writer Eric Rogers stated that this was his favorite episode of the season.\(^17\)\(^18\)
References


External links

- Keeler's theorem and products of distinct transpositions (http://arxiv.org/abs/1204.6086)
- "The Prisoner of Benda" at the Infosphere, the Futurama Wiki.
- "A Prisoner of Benda" (http://tv.msn.com/tv/episode/futurama/a-prisoner-of-benda/) at MSN
- The Infosphere's take on the Futurama theorem (http://theinfosphere.org/Futurama_theorem)
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