Sections 10.1 to 10.3

Annuities or sinking funds formula - used to figure the amount that will be in an account after a constant amount has been deposited over time.

\[ FV = Pmt \left( \frac{1 + \frac{r}{n}}{\frac{r}{n}} \right)^{nt} - 1 \]
We will deposit $100 a month into an account compounding monthly at 4.85% for 15 years as a college fund. How much money will there be in the account and how much will be made from interest?

\[
FV = 100 \left( 1 + \frac{.0485}{12} \right)^{12(15)} - 1
\]

Calculator:

\[
100 \left( (1 + \frac{.0485}{12})^{12 \times 15} - 1 \right) / (.0485/12) = 26,396.70
\]

Interest:

\[
100(12)(15) = 18,000
\]

\[
= 9,396.70
\]

For a vacation, you are depositing $50 weekly into an account for 2 years that compounds weekly at 5.45%. How much is in the account and how much is from interest after the 2 years?

\[
FV = 50 \left( 1 + \frac{.0545}{52} \right)^{52(2)} - 1
\]

\[
50 \left( (1 + \frac{.0545}{52})^{52 \times 2} - 1 \right) / (.0545/52) = 5490.95
\]

\[
I = 50(52)(2) = 5200
\]

\[
= 290.95
\]
For a retirement account, we will be depositing $550 at the beginning of each quarter in an account compounding quarterly at 8.75% for 22 years. How much is in the account and how much is from interest?

\[
FV = 550 \left( 1 + \frac{0.0875}{4} \right)^{4 \times 22} - 1 - 550
\]

\[
= 550 \left( \left( 1 + \frac{0.0875}{4} \right)^{22.4} - 1 \right) / \left( \frac{0.0875}{4} \right) - 550
\]

\[
= \$146,818.04
\]

I = 550(4)(22) = \$48,400

\[
\$98,418.04
\]

IRA - Individual Retirement Account

Regular IRA - excluded from federal income taxes, but taken out when the account is being withdrawn

Roth IRA - NOT excluded from taxes on deposit, but is excluded in withdrawal

Retirement Plans

401(k) - retirement account through a private sector business

403(b) - retirement for public schools or tax exempt organizations
You start at a business with a 401(k) and find the account is compounded monthly and you are set to deposit $150 per month with a company match. The account runs 5.35% and you will be staying at the company for the life of 35 years. How much will you have in the account when you retire? Interest?

\[
FV = Pmt \cdot \frac{1}{\frac{r}{n}} \left(1 + \frac{r}{n}\right)^{nt} - 1
\]

\[
PV = Pmt \cdot \frac{1 - (1 + \frac{r}{n})^{-nt}}{\frac{r}{n}}
\]

Section 10.2
A college student needs $1500 per month while in college for 5 years. They have found an account that pays 6.15% compounding monthly. What lump sum would have to be deposited today to allow the student that freedom?

\[
P = \frac{1500}{1 - \left(1 + \frac{0.0615}{12}\right)^{-12 \cdot 5}} \frac{0.0615}{12}
\]

\[
1500 \left(1 - \left(1 + \frac{0.0615}{12}\right)^{-12 \cdot 5}\right) \frac{0.0615}{12} = 77,301.12
\]

In selling a business, you have two offers. One offer is for a down payment of $25000 and then semiannual payments of $2500 for 5 years. The other offer is for $48000 straight cash. Which is a better deal if you can invest at 3.85% compounding semiannually?

\[
PV = \frac{2500}{1 - \left(1 + \frac{0.0385}{2}\right)^{-2 \cdot 5}} \frac{0.0385}{2}
\]

\[
2500 \left(1 - \left(1 + \frac{0.0385}{2}\right)^{(-2 \cdot 5)}\right) \frac{0.0385}{2} = 22,544.84 + 25000 = 47,544.84
\]
You are purchasing a car for $185 per month and it compounds monthly with 9.45% interest for 48 months. What is the present value of the car?

\[ PV = 185 \left(1 - \left(1 + \frac{.0945}{12}\right)^{-12(48)}\right) \]

\[ PV = \$7370.73 \]

Section 10.3

How much would I have to deposit in an account per month for my child to have $60000 for college in 17 years, if I have an account that pays 7.35% and compounds monthly? How much interest will I make?

\[ 60000 = \frac{60000}{(1 + \frac{.0735}{12})^{12(17)}} - 1 \]

\[ \text{pmt} = \frac{60000}{(1 + \frac{.0735}{12})(1 - \left(1 + \frac{.0735}{12}\right)^{-12(17)}} \]

\[ \text{pmt} = \$148.47 \]

\[ I = 148.47 \times 12 \times 17 = \$30,287.88 \]

\[ \text{I} = \frac{30,287.88}{121,712.12} \]
The business that you work for will be replacing their ventilation system in 7 years. The cost of the project is estimated to be $2.3 million. What amount would have to be deposited quarterly in an account that pays 6.85% compounded quarterly to cover the cost?

\[ 2,300,000 = \text{pmt} \times \left( \frac{(1 + \frac{.0685}{4})^{4(7)} - 1}{\frac{.0685}{4}} \right) \]

\[ 2,300,000 / (((1 + .0685/4)^{4.7} - 1) / (.0685/4)) = \$4705.15 \times 7 \times 4 = \$1811,744.20 \]

\[ I = \$488,255.80 \]

Can we make $1000000 in our life? Over the long life of the stock market, you can guess an average of 10% compounded annually. How much would have to be deposited annually for that to happen if you continue your experiment for 60 years?

\[ 1,000,000 = \text{pmt} \times \frac{(1 + \frac{.10}{1})^{1(60)} - 1}{\frac{.10}{1}} \]

\[ 1,000,000 / (((1 + .10/1)^{1(60)} - 1) / (.10/1)) = \$329,51 \]
Have a good day.