Factoring the Illegal Way

Remember that the standard form of a quadratic expression is \( ax^2 + bx + c \).

Example: \( 6x^2 - 7x - 3 \)

This is difficult to factor because the coefficient of the squared term (a) is not 1. Therefore I remove the 6 by multiplying it with the c term (-3). My new trinomial is

\( x^2 - 7x - 18 \)

Now this trinomial is easily factored into \((x - 9)(x + 2)\).

I did an “illegal move,” and I now need to “undo” it. Since I multiplied by 6 in the first step, to “undo” it, I now divide each constant by 6.

\[
\left( x - \frac{9}{6} \right) \left( x + \frac{2}{6} \right)
\]

I now have a factored form with fractions. That is not acceptable so I first reduce the fractions to lowest terms.

\[
\left( x - \frac{3}{2} \right) \left( x + \frac{1}{3} \right)
\]

The binomials still have fractions that cannot be reduced, so I simply take the denominator of the fraction and squeeze it in front of the x in the binomial, making the denominator the coefficient of x.

\[
\left( x - \frac{3}{2} \right) \left( x + \frac{1}{3} \right)
\]

It works every time!

The only problem I have had with students using this method is they forget to undo the illegal move. You must do both steps!!

This does not work if a is a negative number. Factor out a -1 and then proceed.
Here’s another example:

\[ 2x^2 - 7x - 15 \]

Because a is not 1, I perform the illegal move. **Multiply by 2.**

\[ x^2 - 7x - 30 \]

Factor using the sign clues.

\[ (x - 10)(x + 3) \]

Now I must undo the illegal move. **Divide by 2.**

\[ \left( x - \frac{10}{2} \right) \left( x + \frac{3}{2} \right) \]

Simplify the fractions if I can.

\[ (x - 5) \left( x + \frac{3}{2} \right) \]

Since I still have a fraction, I move the denominator of the fraction in front of the variable.

\[ (x - 5)(2x + 3) \]

**One More Example:**

\[ 12x^2 + 17x + 6 \]

Perform the illegal move. **Multiply by 12.**

\[ x^2 + 17x + 72 \]

Factor using the sign clues.

\[ (x + 8)(x + 9) \]

Undo the illegal move. **Divide by 12.**

\[ \left( x + \frac{8}{12} \right) \left( x + \frac{9}{12} \right) \]

Reduce fractions.

\[ \left( x + \frac{2}{3} \right) \left( x + \frac{3}{4} \right) \]

Remove fractions.

\[ (3x + 2)(4x + 3) \]