Objectives

1. Factor trinomials of the form $x^2 + bx + c$.
2. Factor out a monomial GCF, then factor the trinomial of the form $x^2 + bx + c$.

Vocabulary

Prior Knowledge

New Concepts

1. Factor trinomials of the form $x^2 + bx + c$.

Study these products to see some patterns that might be helpful in factoring trinomials.

$(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$
$(x - 3)(x - 5) = x^2 - 5x - 3x + 15 = x^2 - 8x + 15$
$(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$
$(x - 3)(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 2x – 15$

Notice on the first two rows, the last number is +15 and in the last two rows, the last number is -15. So to find the numbers in the binomials, we have some very helpful sign clues.

If the last sign of the trinomial is + :

- both signs in the binomials will be the same. They will be the same sign as the middle term of the trinomial.
- the numbers in the binomials must be factors of 15 that add to equal the middle term 8.

If the last sign of the trinomial is - :

- the signs in the binomials will be different: one will be + and one will be - . The larger number will have the same sign as the middle term of the trinomial
- the numbers in the binomials must be factors of 15 that subtract to equal the middle term 2.

Example 1: Factor. $y^2 + 12y + 20$

Because the coefficient of the last term is + 20, the signs of the binomials must be the same and since the coefficient of the middle term is + 12, both signs must be + .

$(y + \quad)(y + \quad)$

The numbers in the binomials must be factors of 20 that add (because of the +20) to equal 12: has to be 10 and 2.

$(y + 2)(y + 10)$

Check by foiling to be sure you are correct.

Example 2: Factor. $a^2 - 5a + 6$

Because the coefficient of the last term is +6, the signs of the binomials must be the same and since the coefficient of the middle term is -5, both signs must be - .

$(a – \quad)(a – \quad)$

The numbers in the binomials must be factors of 6 that add (because of the +6) to equal 5: has to be 3 and 2.

$(a - 3)(a - 2)$

Check by foiling to be sure you are correct.

Example 3: Factor. $\mathbf{a^2 - 9a - 22}$

Because the coefficient of the last term is -22, the signs of the binomials must be different and since the coefficient of the middle term is -5, the larger number will be negative.

$(a - \quad)(a + \quad)$

The numbers in the binomials must be factors of 22 that subtract (because of the -22) to equal 9: has to be 11 and 2.

$(a - 11)(a + 2)$

Check by foiling to be sure you are correct

Example 4: Factor $\mathbf{t^2 + t - 20}$

Because the coefficient of the last term is -20, the signs of the binomials must be different and since the coefficient of the middle term is +1, the larger number will be positive.

$(t - \quad)(t + \quad)$

The numbers in the binomials must be factors of 20 that subtract (because of the -20) to equal 1: has to be 4 and 5.

$(t - 4)(t + 5)$

Check by foiling to be sure you are correct.

Example 5: Factor. $\mathbf{y^2 + 2y + 3}$

Because the coefficient of the last term is + 3, the signs of the binomials must be the same and since the coefficient of the middle term is + 2, both signs must be + .

$(y + \quad)(y + \quad)$

The numbers in the binomials must be factors of 3 that **add** (because of the +3) to equal 2. The only factors of 3 are 1 and 3. They subtract to equal 2 but our signs tell us they have to add to equal 2. No such numbers exist so this trinomial cannot be factored. It is a **prime polynomial.**

When factoring a trinomial with two variables, the double variable in the middle term comes from multiplying the outside and inside terms

(__x + __y) (__x + __y)

Outside: __x · __y = __x y
Inside: __y · __x = __x y

When both the first and last terms of a trinomial contain perfect square variables, place the variables in the binomials and then continue to find the numeric coefficients.

Example 6: Factor $x^2 - 2xy - 15y^2$

Because there is both an x^2 and a y^2, place both the x and the y in the binomials.

$(x \quad y)(x \quad y)$

Because the coefficient of the last term is -15, the signs of the binomials must be different and since the coefficient of the middle term is -2, the larger number will be negative.

$(x - \quad y)(x + \quad y)$

The numbers in the binomials must be factors of 15 that subtract (because of the -15) to equal 2: has to be 3 and 5.

$(x - 5y)(x + 3y)$

Check by foiling to be sure you are correct.

ALWAYS look for a GCF. Many difficult looking problems are easy to factor once the GCF is removed. This is a crucial first step as the problems become more complicated!

Example 7: Factor $2k^2y - 18ky + 28y$

GCF: 2y

$2y(k^2 - 9k + 14)$

$2y(k - 2)(k - 7)$

Check by foiling to be sure you are correct.