Physics 211: University Physics Laboratory

Laboratory Manual

Montana State University-Billings
Lab 1
Constant Acceleration

Introduction
In this lab, you will study the nature of the equations of motion for constant acceleration. You will use the simulation of the inclined plane and calculate the acceleration of a block down a frictionless inclined plane using measurements of the block’s position. Once you have calculated the acceleration, you will use the equations of motion to determine the initial velocity necessary to make the block travel up the inclined plane a distance of 2 meters before turning around.

Theory
In order to describe the motion of any object we use an equation of motion which is used to solve for the position as a function of time. In the case of one-dimensional, constant acceleration motion, this equation of motion takes the form of the definitions of acceleration and velocity.

\[ a = \frac{\Delta v}{\Delta t} = \text{constant} \]
\[ v = \frac{\Delta x}{\Delta t} \]

From these definitions, it follows that the most general function \( x(t) \) which produces a constant acceleration is given by:

\[ x(t) = x_0 + v_0 t + \frac{1}{2} at^2. \]

For any situation in which the acceleration of an object is constant, the position will vary as a function of time according to the formula above, and the velocity will be given by:

\[ v(t) = v_0 + at. \]

Using these two formulas, it is possible to derive all of the one-dimensional, constant acceleration equations in your book. The motion of a block down an inclined plane is an example of such motion.

Procedure
Start up the PEARLS software and select the Mechanics package from the Topics list and select Inclined Plane from the Simulations list. The Inclined Plane simulation is set up to do a variety of different simulations including friction. We will be studying the frictionless case in this lab, so the first thing you do is set the Static Friction Coefficient to zero. The default setting for the angle is 35° — pick a different angle between 5° and 20°. At this point, you have the experimental apparatus set up. Now you will want to take some data. Click on the button with a graph icon. You should be presented with a range of choices for graphing. Choose time for the x-axis and displacement for the y-axis and click OK. You should see a graph floating over the inclined plane simulation. You are now ready to run the experiment. Click on the play button and you should see the block
start to slide down the inclined plane. You should also see a parabola begin to grow on
your floating graph. Stop the simulation by hitting the pause button. You can now reset
the simulation by hitting the “t = 0” button. At this point you have simulated taking
position measurements of the block at repeated time intervals. The data is presented on
the graph. However, the data has also been recorded as a list of positions at given times.
This is the data that you will need to use to determine the block’s velocity and
acceleration. To extract the list of data, click on the disk icon at the bottom of the graph.
You will be asked where to save graph data. Choose the “desktop” button and then
rename the graph data “position1”. Click OK. You should now have a document on the
desktop titled “position1”. Go to the desktop (ask if you don’t know how) and double
click on “position1” you should have a program called “SimpleText” open up with a page
containing two columns of data. The first column is time and the second column is
position. Pick a cluster of three consecutive times and positions from near the beginning
of the data stream and record them. Next pick another cluster of three consecutive times
and positions from near the end of the data stream and record them. You can now
calculate the velocity and acceleration from near the beginning and near end of the data.

Consider one cluster of three data points:

\[
\begin{array}{cc}
    t_1 & x_1 \\
    t_2 & x_2 \\
    t_3 & x_3 \\
\end{array}
\]

The velocity is given by \( v = \frac{x_2 - x_1}{t_2 - t_1} \) and is occurs at approximately \( t = \frac{1}{2}(t_1 + t_2) \)
(actually this is exact for constant acceleration and approximate for other types of
motion). Calculate the velocity between times \( t_1 \) and \( t_2 \) and between times \( t_2 \) and \( t_3 \) so that
you have two velocities separated by a known time interval. Use these to calculate the
acceleration which is given by \( a = \frac{v_{23} - v_{12}}{t_{23} - t_{12}} \). Record all calculations and all data used in
the calculations. Repeat this procedure for the cluster of data near the end of the data
stream. Are the calculated accelerations the same? If not, are they close? Before
continuing to the next part of the lab, it is useful to perform a check to make sure the data
is reasonable. As we will learn in the chapters on Newton’s laws, the acceleration down
an inclined plane is given by \( a = g \sin \theta \) where \( g = 9.8 \text{ m/s}^2 \). Using the angle from your
simulation calculated the expected acceleration. Is it close to the value you calculated
using the data points? If not, start over.

Using the formulas for motion with constant acceleration determine the initial velocity
needed to have the block rise to a position 2 meters up the ramp before beginning to slide
back down the ramp. Show all your work. Next, return to the simulation and click on the
t=0 button to reset the block at its initial position. Click on the eraser icon underneath
your displacement vs. time graph to erase your original data. Next, scroll down until you
see the Initial Speed control. Input your calculated initial speed (remember, uphill is
negative), and then scroll up to run the simulation. After the block has risen to its
maximum height and then slide back down past its initial position, stop the simulation.
Extract the new data by clicking on the disk icon under the graph and saving the data as position2 on the desktop. Open this data file and determine the highest position the block reached on the inclined plane. Record this value \((x_m)\) and the time \((t_m)\) at which it occurred. Is \(x_m\) equal to 2 meters? If not, is it close? Use \(t_m, v_0,\) and \(a\) to calculate the position at \(t_m\), is it near 2 meters?

To determine how much your expected values differ from the measured values, it is useful to calculate the “percent deviation” of the expected value from the measured value. Although the distinction is a little fuzzy in a simulated lab, we generally consider the measured value as reality and we compare the calculated value against reality. Consequently, if \(M_{ex}\) is an experimental measurement and \(M_{th}\) is a theoretical calculation of what that measurement should be, then the percent deviation is calculated as

\[
\% \text{ dev.} = \frac{|M_{ex} - M_{th}|}{M_{ex}}
\]

and it measures how well our theory stands up to reality. Calculate the percent deviation of your experimental value of \(a\) with the one calculated using \(a = g \sin \theta\), and the percent deviation of your experimental value of \(x_m\) with the calculated value of 2 m.

When you have finished the lab, clean up after yourself by deleting the datafiles you have saved to the desktop and quitting all open applications (SimpleText, PEARLS, and Inclined Plane).
In this lab, we will use the equations for projectile motion to determine the initial velocity of a ball fired from a spring-loaded gun, which we will then use to predict the range of the ball when the gun is elevated. The gun is placed on a lab table so that it will fire a ball horizontally off of the table. From our knowledge of projectile motion, we know that it will follow a parabolic path to the floor. If we know the height of the table and the range of the ball, we can work the equations for projectile motion backwards to find the initial velocity of the ball as it leaves the gun. If these equations hold true, then we should be able to predict where the ball will land if the gun is elevated so that the initial velocity is not horizontal. Thus, this experiment is going to test the validity of the vector equation:

\[
\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2.
\]

**Procedure:**

Set the gun up on the edge of a lab table and fire the gun once to get an idea for the range of the ball. Next, tape a piece of paper to the floor in the approximate location of where the ball landed. Fire the gun again. If the ball lands on the paper, it will leave a mark. Do this five times without moving the gun or the paper. Measure the horizontal distance from the end of the gun to the marks on the paper and take the average of the five readings. Record this value and the largest deviation which your individual measurements make from the average:

\[
R_{av} =
\]

\[
\Delta R =
\]

Measure and record the height of the gun off of the floor:

\[
h =
\]

Knowing that the ball has no initial vertical component of velocity, we can calculate the time required for it to drop from the height \( h \). Show this calculation and record your answer:
t =

Knowing the range of the ball and the time that it spends in the air, we can calculate the initial velocity. Show this calculation and record your answer:

\[ v_o = \]

At this point, we now know the initial velocity of the ball as it leaves the gun. If the front of the gun is elevated so that it makes an angle \( \theta \) with the horizontal, we have a generic projectile motion problem in which the height (h), the magnitude of the initial velocity (\( v_o \)), and the angle (\( \theta \)) are known. We can now calculate the expected range. To find the angle \( \theta \), measure the height that the front of the gun is elevated by, and the distance between the feet of the gun. The angle can be found using trigonometry from these two measurements. Measure h and \( \theta \) for the new set-up:

\[ h = \]
\[ \theta = \]

Using the \( v_o \) from the first part, calculate the expected range and record this value:

\[ R_{ex} = \]

Next, tape the paper in the appropriate region where you expect the ball to land and fire the gun five times. Find the average value and record the largest deviation which your individual measurements make from the average:

\[ R = \]
\[ \Delta R = \]

Is this deviation comparable to the one found in the first part? (If not, go back and do it again). Does \( R_{ex} \) fall within the range given by \( R \pm \Delta R \)? If not, explain why you think it didn't.
Whenever a body slides along another body, a resisting force appears which is called the force of friction. This is a very important force and serves many useful purposes. A person could not walk without it, or a car could not propel itself along a road without the friction between the tires and the road. On the other hand, friction is sometimes very wasteful. It reduces the efficiency of machines, work must be done to overcome it, and this work is wasted as heat. The purpose of this experiment is to study the laws of sliding friction and to determine the coefficient of friction between two surfaces.

**THEORY:**

Friction is the resisting force encountered when one surface slides, or tends to slide, over another; this force acts along the tangent to the surfaces in contact. The size of this frictional force depends only on the nature of the materials in contact (their roughness or smoothness) and on the normal force, it does not depend upon the area of contact. It is found experimentally that the force of friction is directly proportional to the normal force. The constant of proportionality is called the coefficient of friction. The coefficient of friction ($\mu$) is defined to be the ratio of the force of friction to the total normal force between the surfaces. Thus:

$$\mu = \frac{F_f}{N} \quad \text{or} \quad F_f = \mu N.$$

Note that the value of $\mu$ depends upon the surfaces in contact and therefore is different for different substances. Thus, the coefficient of friction is given for substance A on substance B -- for example, $\mu = 0.19$ for pine on particle board. We will determine the coefficient of friction for two particular substances using two different methods, and then we will show that $\mu$ is independent of the normal force.

**PROCEDURE:**

If a body is placed on an inclined plane which is not frictionless, then the angle at which the body will slide down the plane at constant velocity (i.e. with no acceleration) is related to the coefficient of friction by $\mu = \tan(\theta)$ where $\theta$ is called the "angle of repose".
Draw a free-body diagram on the next page and prove that this is true using Newton's second law:

Take the frictional surface and place the brick on it. Begin tilting the surface while tapping the brick. When you reach the angle of repose, the brick will begin sliding down the surface at constant velocity. Record this angle, and calculate the coefficient of friction:

\[ \theta = \]  
\[ \mu = \]  

If a body is placed on a flat surface which is not frictionless and connected to a hanging weight by a pulley and string, then the body will move at a constant velocity when the frictional force is balanced by the tension in the string. Draw a free-body diagram of this situation and, using Newton's second law, show that the frictional force must be equal to the hanging weight if the acceleration is to be zero:

Take the frictional surface and lay it level. Connect the hanging weight to the brick via the pulley. Increase the hanging weight by adding mass to it while tapping the brick. When the hanging weight is equal to the frictional force, the brick will begin to slide down the frictional surface at constant velocity. Record this weight (which is also the frictional force). Repeat this experiment five times, each time adding 50 grams to the mass of the brick by placing additional 50 g masses on top of the brick. Record the frictional force for each case. Find the normal force exerted by the frictional surface on
the brick by finding the weight of the brick. Record this value. The coefficient of friction is then given by:

\[ \mu = \frac{F_f}{N}. \]

Calculate \( \mu \) for each of the six trials (six different values of \( N \)). After completing the above procedure, you should have filled in the following data table:

<table>
<thead>
<tr>
<th>M of brick</th>
<th>added M</th>
<th>N=Mg</th>
<th>hanging m</th>
<th>( F_f = mg )</th>
<th>( \mu = F_f/N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>100</td>
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<tr>
<td>150</td>
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<tr>
<td>200</td>
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</tr>
<tr>
<td>250</td>
<td></td>
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</tr>
</tbody>
</table>

Is the value of \( \mu \) dependent upon the normal force? If it is, should it be?
Lab 4
Uniform Circular Motion

In order for a body to move in a circle or travel along a circular path, there must be a net outside force applied on that body. Newton's first law requires this. In addition, using a bit of geometry, we can show that this applied force must constantly change its direction so as to always point towards the center of the circle. Such a force is called a centripetal force. Whenever you see anything moving in a circle, you may rest assured that somewhere there is an agent supplying the necessary centripetal force to allow for such "unnatural" motion. (Remember, straight line motion is "natural" motion.) It might be helpful for you to analyze a few cases of circular motion to be sure that you can find the source of the force responsible for this peculiar motion. the purpose of this experiment is to study uniform circular motion and to compare the measured value of the centripetal force with the value one would expect from theoretical considerations.

Theory:

If a body is moving with uniform speed in a circle of constant radius r, it is said to be moving in uniform circular motion. Even though the speed is constant, the velocity is continually changing, since the direction of the motion is constantly changing. Thus such a body is accelerating. Recall from class that the direction of the acceleration is always toward the center of the circle and the magnitude of the acceleration is given by

$$a = \frac{v^2}{r}$$

In the cgs system of units, v is the speed of the body in centimeters per second and r is the radius of the path in centimeters. Note: The direction of the acceleration is continuously changing as the body moves around the circle.

We know from Newton's second law that accelerations don't just happen all by themselves. There must be a net outside force acting on the object in order for it to accelerate. Furthermore, if the object has a mass of m and experiences an acceleration a, then the size of the force must be numerically equal to m times a. Because this force is directed inward towards the center of the circle it is sometimes called a centripetal force. Hence, the size of the necessary force for an object of mass m to move in a circle of radius r with a speed v is given by:
\[ F_c = \frac{mv^2}{r} \]

If \( m \) is measured in grams and \( r \) and \( v \) are as before, then \( \frac{mv^2}{r} \) will have the units of dynes. Whatever outside agent is supplying this \( F_c \) will feel (by Newton's third law) a "center fleeing" force which acts on the agent, not on the mass, and hence it does not affect the motion of the mass at all. It may be important to the agent doing the pushing, but then we are not interested in anything but the mass and so we'll let the agent sweat the centrifugal force out and we won't worry about it. The centripetal force can also be expressed in terms of the frequency of rotation of the mass since

\[ v = (2\pi)f \]

where \( 2\pi r \) is the circumference of the circular path and \( f \) is the frequency of rotation (number of revolutions per second). Thus:

\[ F = \frac{m(2\pi f)^2}{r} = 4\pi^2 f^2 mr \]

where \( F \) is the centripetal force in dynes.

**Procedure:**

In this experiment, we will test this theory by spinning a mass in uniform circular motion and then measuring the force required to produce this motion. In addition, we will measure the radius of the circle, the mass of the object, and the frequency of rotation. Thus, we can calculate \( F \) using the above formula and compare the calculated value with the measured value. In the apparatus used for this experiment, the force of a stretched spring supplies the centripetal force required to keep a mass \( m \) moving with uniform circular motion in a circle of fixed radius. Think for a moment. If the linear speed of the mass should increase for some reason, then the spring at the original stretch can no longer supply the necessary force. Something has to give and in this case, the mass creeps out (\( r \) increases), the spring stretches more and therefore pulls harder, and everything settles into a new circle with a large radius than before.

In this apparatus, when a critical speed is reached (the mass moves in a circle of known radius) an indicator will pop up. The speed is kept constant and is measured by means of a revolution counter and a stopwatch. The force required to stretch the spring can be measured by removing the spinning apparatus from the machine and hanging weights off of the spring until it stretches the same amount.
First, adjust the speed of the rotator and keep it constant at the critical speed. Measure the frequency of rotation with the revolution counter and a stopwatch. First record the reading of the counter. At the proper instant engage the counter and start the stopwatch. To perform the experiment more conveniently, it is a good plan to let one observer pay strict attention to the proper adjustment of the speed. The other observer can obtain the value of the frequency by manipulating the stopwatch and the revolution counter. At the end of exactly one minute, disengage the counter and record the reading. Do this procedure 3 times to fill out the data table given:

<table>
<thead>
<tr>
<th>Time Interval Seconds</th>
<th>Readings of the Revolution Counter At the Beginning</th>
<th>Frequency Rotation rev/sec</th>
</tr>
</thead>
</table>

<p>| | | |</p>
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</tbody>
</table>

Next, remove the apparatus, suspend it from a supporting stand, and determine the weight necessary to stretch the spring an equivalent amount. Remember to include the weight of the rotating mass in computing the total force needed to stretch the spring. Finally, measure the radius of rotation of the mass by measuring the distance from the axis of rotation to the center of the mass. **Do this before removing the weight.** Record this information in the following data table and do the necessary calculations to fill out the remainder of the table.

**Data**

- Mass of the rotating mass.
- Weight of the rotating mass.
- Total weight necessary to stretch the spring.
- Radius of rotation.
- Average frequency of rotation.
- Centripetal force calculated from the theory.
- Measured value of the centripetal force.
Find the "percent discrepancy" between the theoretical value of the centripetal force ($F_{Th}$) and the measured value ($F_{Me}$) which is given by:

$$\%\text{disc} = \frac{|F_{Th} - F_{Me}|}{F_{Me}} =$$
The principle of conservation of momentum follows directly from Newton's laws of motion. According to this principle, if there are no external forces acting on a system containing several bodies, then the momentum of the system remains constant. In this experiment the principle is applied to the case of a collision, using a ballistic pendulum. A ball is fired by a gun into the bob of a pendulum. The initial velocity of the ball is determined in terms of the masses of the ball and pendulum bob and the height which the bob rises after impact.

Theory:
The momentum of a body is defined as the product of the mass of the body and its velocity. Newton's second law of motion states that the net force acting on a body is proportional to the time rate of change of momentum. Hence, if the sum of the external forces acting on a body is zero, the linear momentum of the body is constant. This is essentially a statement of the principle of conservation of momentum. Applied to a system of bodies, the principle states that if no external forces act on a system containing two or more bodies then the momentum of the system does not change.

In a collision between two bodies, each one exerts a force on the other. These forces are equal and opposite, and if no other forces are brought into play, the total momentum of the two bodies is not changed by the impact. Hence the total momentum of the system after the collision is equal to the total momentum before impact. During the collision the bodies become deformed and a certain amount of energy is used to change their shape. If the bodies are not perfectly elastic, they will remain permanently distorted, and the energy used up in producing the distortion is not recovered.

Inelastic impact can be illustrated by a device called a "ballistic pendulum", which is sometimes used to determine the speed of a bullet. If a bullet is fired into a pendulum bob and remains imbedded in it, the momentum of the bob and bullet just after the collision is equal to the momentum of the bullet just before the collision. This follows from the law of conservation of momentum. The velocity of the pendulum before collision is zero, while after the collision, the pendulum and the bullet move with the same velocity. Hence the momentum equation gives:

$$mv = (M + m)V$$

where m is the mass of the bullet, v is the velocity of the bullet just before impact, M is the mass of the pendulum bob, and V is the common velocity of the pendulum bob and bullet just after the collision.
As a result of the collision, the pendulum with the imbedded bullet swings about its point of support, and the center of gravity of the system rises through a vertical distance $h$. From a measurement of this distance it is possible to calculate the velocity $V$. The kinetic energy of the system just after the collision must be equal to the increase in potential energy of the system as the pendulum reaches its highest point. This follows from the law of conservation of energy; here we assume that the loss of energy due to friction is negligible. The energy equation gives

$$(\frac{1}{2})(M + m)V^2 = (M + m)gh$$

where $h$ is the vertical distance through which the center of gravity of the system rises. The left hand side of the equation represents the kinetic energy of the system just after the collision and the right hand side represents the change in potential energy of the system. Solving the above equation for $V$ one obtains:

$$V = \sqrt{2gh}.$$ 

By substituting this value of $V$ and the values of the masses $M$ and $m$ in the momentum equation, it is possible to calculate the velocity of the bullet before the collision. Thus,

$$v = \frac{M + m}{m} \sqrt{2gh}.$$ 

**Procedure:**

Get the gun ready for firing. Release the pendulum from the rack and allow it to hang freely. When the pendulum is at rest, pull the trigger, thereby firing the ball into the pendulum bob. This will cause the pendulum with the ball inside it to swing up along the rack where it will be caught at its highest point. Measure the change in height of the center of mass of the pendulum bob. Repeat this procedure five times and record all five values of $h$ and the average $h_{av}$.

<table>
<thead>
<tr>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
<th>$h_5$</th>
<th>$h_{av}$</th>
</tr>
</thead>
</table>

Remove the pendulum bob from the apparatus and find its mass. Find the mass of the ball. Record these values:

$$M =$$

$$m =$$
Using $h_{av}$ in centimeters, $M$ and $m$ in grams, and $g$ in centimeters per second, calculate the speed of the ball.

$$v =$$

Using your calculated values for $v$ and $V$ to find the amount of energy expended in embedding the ball in the pendulum bob.

$$\Delta E =$$
In this lab, we are going to study the concept of simple harmonic motion as demonstrated by the motion of a simple pendulum to determine the acceleration of gravity, \( g \).

The best example of simple harmonic motion (SHM) is the oscillatory motion exhibited by a mass on a spring. We know that the period of oscillation of a mass on a spring, i.e. the time required for the mass to complete one oscillation is given by

\[
T = 2\pi \sqrt{\frac{m}{k}},
\]

where \( m \) is the mass of the object and \( k \) is the spring constant, or, from Hooke’s law,

\[
k = \frac{\text{restoring force}}{\text{displacement}}.
\]

The motion of a simple pendulum is also an example of SHM. However, in the case of the pendulum, the restoring force is the weight component of the pendulum’s mass, or pendulum bob, tangential to the circular path of the pendulum’s motion. Also, the displacement of the pendulum is the arc length traveled by the bob from its maximum displacement to the lowest point, or equilibrium position, of its path.

Procedure:

1. Draw a free-body diagram of the pendulum of length \( L \) with the bob of mass \( m \) at an angular displacement of \( \theta \) from its equilibrium position.

2. What is the net force acting on the bob in the direction tangential to the bob’s path of motion?
3. What is the arc length, or displacement, of the pendulum when the angular displacement of the bob is $\theta$?

4. What is the theoretical period of the pendulum? Use the small angle approximation that $\sin \theta \sim \theta$.

Experiment:

1. Describe an experiment to determine $g$ from the expression for the period of a pendulum that was derived above.

2. Perform the experiment, record and plot the experimental data, and determine $g$. Attach your data on additional sheets of paper.

3. Compare your value of $g$ with the accepted value. What is the percent error?
Lab 7
Archimedes' Principle

Buoyancy is the name given to the ability of a fluid to sustain a body floating in it, or to diminish the apparent weight of a body immersed in it. The size of the buoyant force was discovered by Archimedes to be exactly equal to the weight of the fluid displaced by the object. In this experiment, we will use Archimedes' Principle to determine the specific gravity (and also the density) of: 1) a solid which is heavier than water, 2) a solid which is lighter than water, and 3) a fluid which is not water.

Theory:
Archimedes' Principle states that the apparent loss of weight of a body immersed in a fluid is equal to the weight of the fluid displaced. This means that the difference in weight of an object when it is weighed in air and then weighed while submerged in a liquid is equal to the weight of the amount of liquid which would occupy the same volume as the object. Let's specialize to water as our liquid since we know that water has a density of $\rho = 1 \text{ g/cm}^3$. If the weight in air of an object which has a volume $V$ is $W$, and the weight in water is $W_1$, then the difference in weights will give the weight of an equivalent volume of water:

$$W - W_1 = W_{H_2O}.$$ 

Thus, the specific gravity of the object can be found to be:

$$S = \frac{W}{W - W_1}$$

since

$$\rho = \frac{W}{V}, \quad \rho_{H_2O} = \frac{W_{H_2O}}{V} \quad \text{and}$$

$$S = \frac{\rho}{\rho_{H_2O}} = \frac{W/V}{W_{H_2O}/V} = \frac{W}{W_{H_2O}} = \frac{W}{W - W_1}.$$ 

To find the specific gravity of a liquid other than water, weigh an object submerged in the liquid. This weight will be called $W_2$, and so $W - W_2$ is the weight of
a volume $V$ of the liquid. Following the same argument as above, it can be shown that the specific gravity of the liquid is:

$$S = \frac{W - W_2}{W - W_1}.$$ 

Finally, to find the specific gravity of an object which floats in water, we cannot simply weigh it in water (since it will not completely submerge). Therefore, we have to tie a sinker to the object to keep it submerged but we also then have to be sure to subtract off the weight of the sinker. To do this, we first weigh the object with the sinker attached, but arrange it so that the object is out of the water and the sinker is completely submerged. This weight we call $W_3$. Next, we weigh the object with the sinker attached while both are submerged. This weight is $W_4$. Now, $W_3$ is the weight of the object in air plus the weight of the sinker in water, and $W_4$ is the weight of the object in water plus the weight of the sinker in water. The weight of the sinker in water cancels out when these two weights are subtracted, so the weight of an equivalent volume of water is simply:

$$W_3 - W_4 = W_{H_2O},$$

and the specific gravity of the object is:

$$S = \frac{W_0}{W_3 - W_4}$$

**Procedure:**
1. Weigh the metal cylinder in air. Record this as $W$.
2. Weigh the metal cylinder in water. Record this as $W_1$.
3. Weigh the metal cylinder in salt water. Record this as $W_2$.
4. Weigh the wooden cylinder in air. Record this as $W_0$.
5. Weigh the wooden cylinder with the sinker attached and the sinker alone immersed in water. Record this as $W_3$.
6. Weigh the wooden cylinder and sinker both submerged in water. Record this as $W_4$. 

Use the above measurements and the formulae given above to fill out the following data table:

\[ W = \]
\[ W_1 = \]
\[ W_2 = \]
\[ W_3 = \]
\[ W_4 = \]
\[ S_{\text{metal}} = \]
\[ S_{\text{liquid}} = \]
\[ S_{\text{wood}} = \]
Lab 8
Simple Harmonic Motion: Mass on a Spring

Theory:

Simple harmonic motion is one of the most common types of motion found in nature, and its study is therefore very important. Examples of this type of motion are found in all kinds of vibrating systems, such as water waves, sound waves, the rolling of ships, the vibrations produced by musical instruments and many others. The archetype for simple harmonic motion is a mass on the end of a spring.

One of the criteria for determining whether a system will produce simple harmonic motion is that the force exerted by the system on a mass \( m \) is proportional to the displacement of the mass from its equilibrium position and that the force points back toward the equilibrium position. For a spring, this force is

\[
F = -k\Delta x
\]

where \( k \) is the spring constant of the spring. This force then gives the following relationship between the acceleration and the position of the mass,

\[
a = -\left(\frac{k}{m}\right)\Delta x.
\]

From the above equation, it can be seen that the differential equation describing the motion of a mass on a spring is

\[
\frac{d^2x}{dt^2} + \frac{k}{m} x = 0
\]
Problem 1:
In the space below, derive an expression for the period of the pendulum by assuming a solution for $x(t)$. You will note that the period is independent of the amplitude of the oscillation. This is true as long as the oscillation is not so big that it starts to permanently deform the spring and the force is no longer described by $F = -k\Delta x$.

Problem 2: We will test your calculation by measuring the spring constant of a spring, the mass of an object oscillating on the end of the spring, and the period of the mass' oscillation. The measured period should be equal to the value calculated using the above formula and the measured values of $m$ and $k$.

Procedure:
We will first measure the spring constant, $k$, by hanging a mass from the end of the spring and allowing the mass to come to equilibrium. At equilibrium, the force of gravity pulling down on the mass will be canceled by the force of the spring pulling up on the mass. Therefore,

$$mg = k\Delta x$$
so

\[ k = \frac{mg}{\Delta x}. \]

Since \( \Delta x \) is equal to the displacement from the equilibrium position of the spring when it has no mass attached to it, we need to first find the equilibrium position of the spring. Hang the spring from a support pole and measure the height of the bottom of the spring from the table. Record this position as the equilibrium position, \( x_o \).

\[ x_o = \]

Now, hang five different masses from the spring and record the mass (m) and the height of the bottom of the spring from the table (h). Calculate the displacement (\( \Delta x = x_o - h \)) and record this for each mass.

<table>
<thead>
<tr>
<th>m (kg)</th>
<th>mg (N)</th>
<th>h (meter)</th>
<th>( \Delta x = x_o - h ) (meter)</th>
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The spring constant \( k \) is the slope of the line found by plotting \( mg \) as a function of \( \Delta x \). Find \( k \) by plotting \( mg \) along the y axis and \( \Delta x \) along the x axis. Draw a best fit line through these data points and then find the slope of the line. This is \( k \) or the spring constant. Record \( k \) and the units associated with it.

\[ k = \]

We will now investigate the relationship between the period of oscillation of the spring and the mass of an object attached to the spring. To do this, hang a known mass (m) on the end of the spring. The total mass suspended by the spring is this mass plus the mass of the spring \( m_s \). Measure the time (t) required for 10 oscillations. Repeat this procedure three times, and record your results in the following data table.
Using the data you have gathered, do the calculations required to fill out the rest of the data table. For each trial, calculate the percent discrepancy between $T_{\text{exp}} = t/10$ and $T_{\text{theory}}$. Record these values below.

<table>
<thead>
<tr>
<th>m</th>
<th>t</th>
<th>$M = m + m_s$</th>
<th>$T_{\text{exp}} = t/10$</th>
<th>$T_{\text{theory}}$</th>
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<table>
<thead>
<tr>
<th>$T_{\text{exp}}$</th>
<th>$T_{\text{theory}}$</th>
<th>% Difference</th>
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Using the data you have gathered, do the calculations required to fill out the rest of the data table. For each trial, calculate the percent discrepancy between $T_{\text{exp}} = t/10$ and $T_{\text{theory}}$. Record these values below.
Lab 9
Resonance in Closed Tubes

The velocity with which a sound wave travels in a substance may be determined if the frequency of the vibration and the length of the wave are known. In this experiment the velocity of sound in air will be found by using a tuning fork of known frequency to produce a wavelength in air which can be measured by means of a resonating air column.

Theory:
If a vibrating tuning fork is held over a tube, open at the top and closed at the bottom, it will send air disturbances, made up of compressions and rarefactions, down the tube. These disturbances will be reflected at the closed end of the tube. If the length of the tube is such that the returning disturbances are in phase with those being sent out by the tuning fork, then resonance takes place. This means that the disturbances reinforce each other and produce a louder sound.

Thus, when a tuning fork is held over a tube closed at one end, resonance will occur if standing waves are set up in the air column with a node at the closed end and a loop (or anti-node) near the open end of the tube. This can take place if the length of the tube is very nearly an odd number of quarter wave lengths of the sound waves produced by the fork. Hence resonance will occur when \( L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \text{ etc.} \), where \( L \) is the length of the tube and \( \lambda \) is the wave length of the sound waves in air. N.B.: The center of the loop is not exactly at the open end of the tube, but is outside of it by a small distance which depends on the wave length and on the diameter of the tube. However, the distance between successive points at which resonance occurs, when the length of the tube is changed, gives the exact value of a half wave length.

The relation between the velocity of sound, the frequency, and the wave length is given by the equation:

\[
v = f\lambda.
\]

The velocity can be calculated from the above equation if the wave length and the frequency are both known. The velocity of sound in air depends on the temperature of the air by:

\[
v = \sqrt{\frac{\gamma RT}{M}}.
\]
Around room temperature, this equation can be approximated by:

\[ v = (331.4 + 0.6T) \text{ m/s} \]

where \( T \) is measured in °C.

**Procedure:**

Adjust the level of the water in the glass tube by raising the supply tank, until the tube is nearly full of water. Hold the tuning fork about 2 cm. above the tube in such a manner that the prongs will vibrate vertically. Start the tuning fork by striking it gently with a rubber mallet (or your knee) -- **do not smack it against the edge of the table**. Slowly lower the water level while listening for resonance to occur. At resonance, there is a sudden increase in the intensity of the sound at the instant the air column is adjusted to the proper length. When the approximate length for resonance has been found, run the water level up and down near this point until the position for maximum sound is found. Measure and record the length of the resonating air column to the nearest millimeter. Make two additional determinations of this length by drastically changing the water level and relocating the position for maximum sound again. Record these two readings also. Lower the water level and repeat this procedure to find the second location at which resonance occurs. Again make three independent determinations of this length and record the readings to the nearest millimeter. Repeat this for a tuning fork with a different frequency. Record the temperature of the room and the frequency of each tuning fork. Use this information to fill out the appropriate positions in the data table. The wavelength of the sound waves can be calculated by finding the difference between the length of the tube at the first position of resonance (\( L_1 \)) and the length at the second position of resonance (\( L_2 \)). This gives one half of a wavelength:

\[ L_2 - L_1 = \frac{\lambda}{2} \]

Using the averages from the data table, calculate the wavelength and record these values in the data table. Finally, calculate the experimental value for the speed of sound in air (\( v = f\lambda \)) from the data for each tuning fork and enter these numbers in the data table.

<table>
<thead>
<tr>
<th>( L_1 )</th>
<th>( L_2 )</th>
<th>( \lambda )</th>
<th>( f )</th>
<th>( v )</th>
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</table>
Using the measured room temperature, calculate the theoretical value of the speed of sound in air from the equation:

\[ \nu = (331.4 + 0.6T) \text{ m/s} . \]

Record this value and compare it with the average of the two experimentally determined values. What is the percent discrepancy?